

Extended Model of Cavity Expansion Theory for Evaluating Skin Friction of Tapered Piles in Sands



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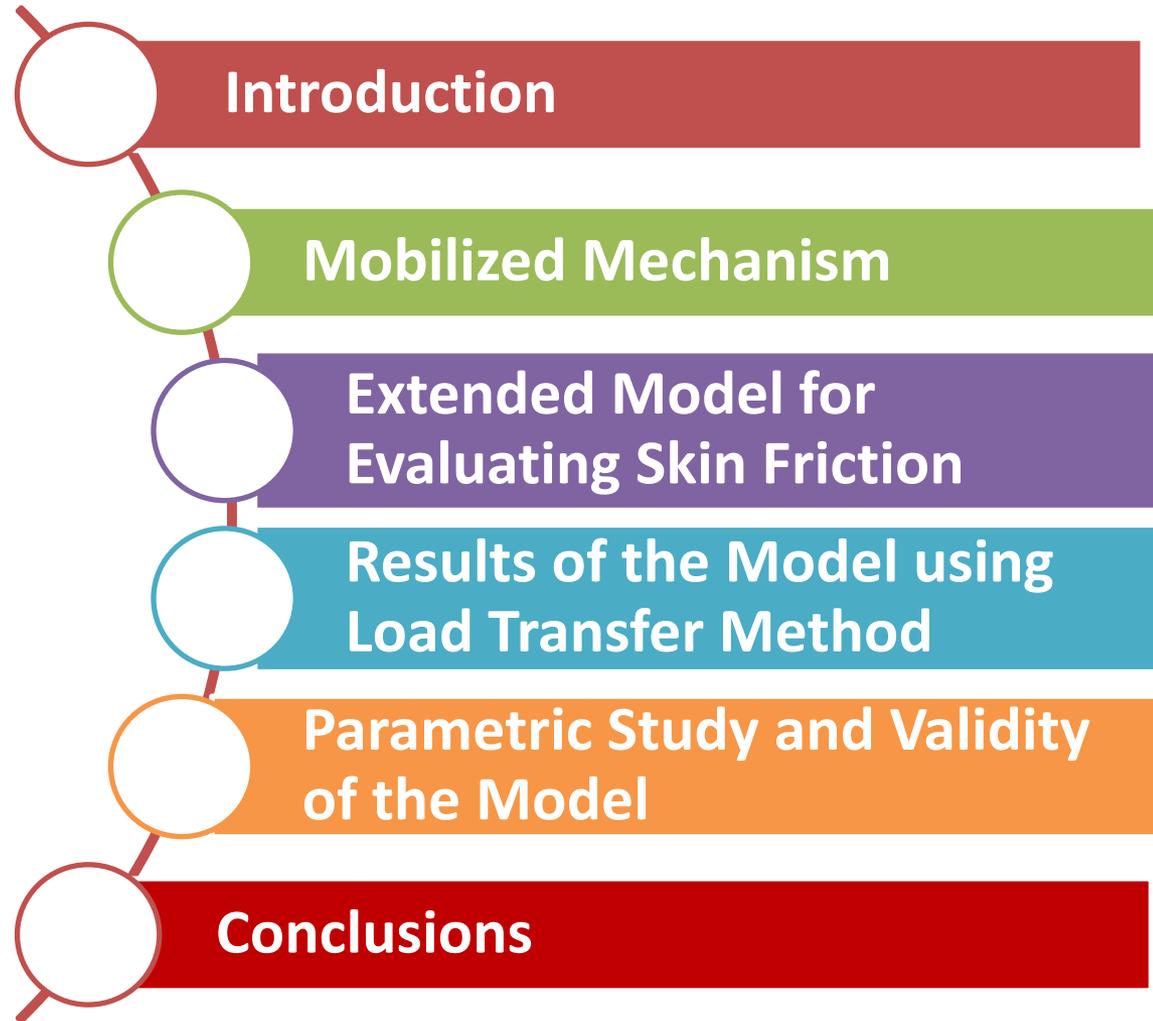
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Outline of the Presentation





Introduction



- **Skin friction** and **radial stress** are highly influenced by **tapered piles** with compared to conventional piles.
- A small increase in the degree of tapering can achieve higher skin friction.
- The **mobilized mechanism** demonstrates a **good pressure effect** when penetrated downward in a frictional mode for sands.
- The **tapering** and **wedging effects** are responsible for *increasing* the normalised skin friction and normalized lateral stresses.



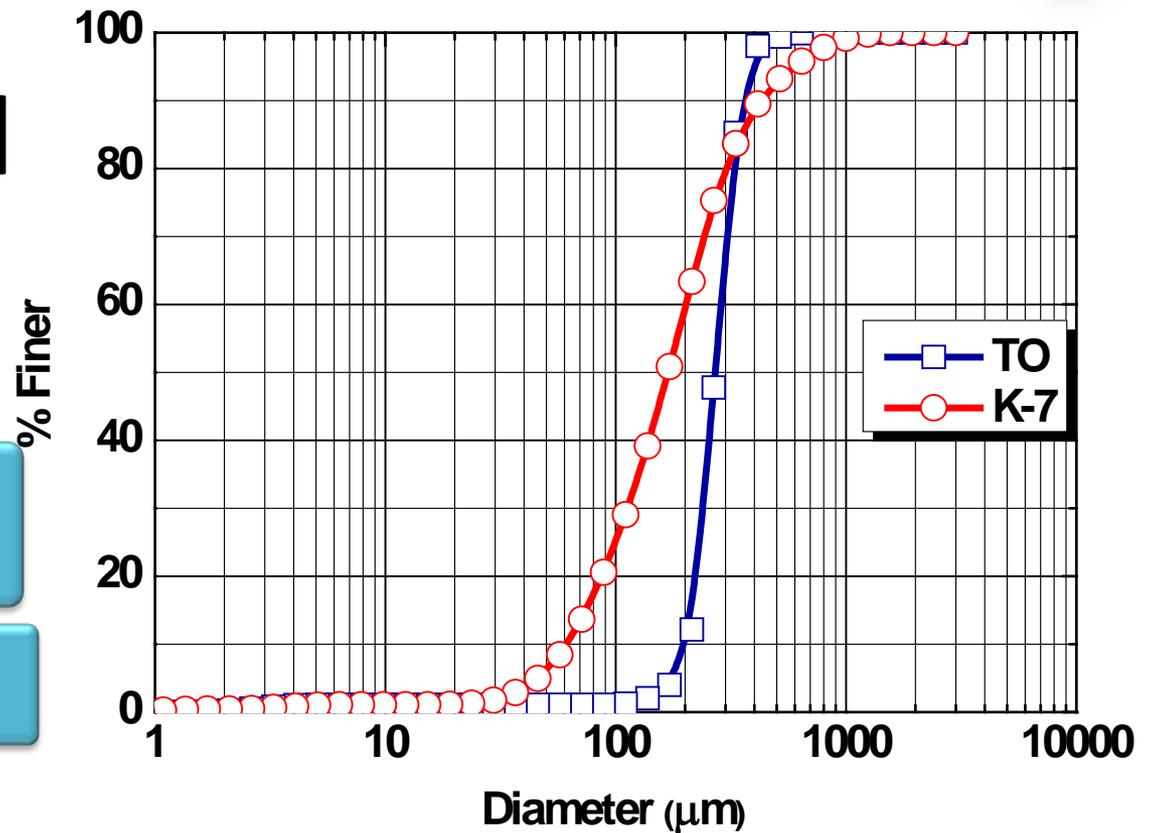
- May be due to a lack of awareness of their basic existence together with a lack of modern and reliable analytical methods, very few researches have been carried out.
- A number of experts in geotech proposed theoretical methods using a **cavity expansion theory**.
- **Vesic` (1972)**: Used to solve an infinite soil body by keeping **volume change** at the same soil at **same density**.



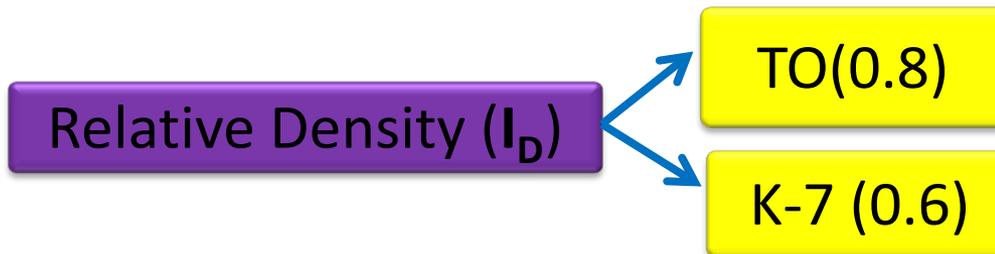
- **Hughes et al. (1977):** Assumed the small elastic deformation in the plastic zone when a limiting value of stress ratio reached after elastic deformation.
- **Carter et al. (1986):** Approximates a steady state deformation mode at *very large deformations* for small deformation problems.
- **Yu and Houlsby (1991):** The most closed form and complete solution for *large straining* condition for cylindrical cavity expansion in an ideal elastic-plastic model.

Materials used

Soils



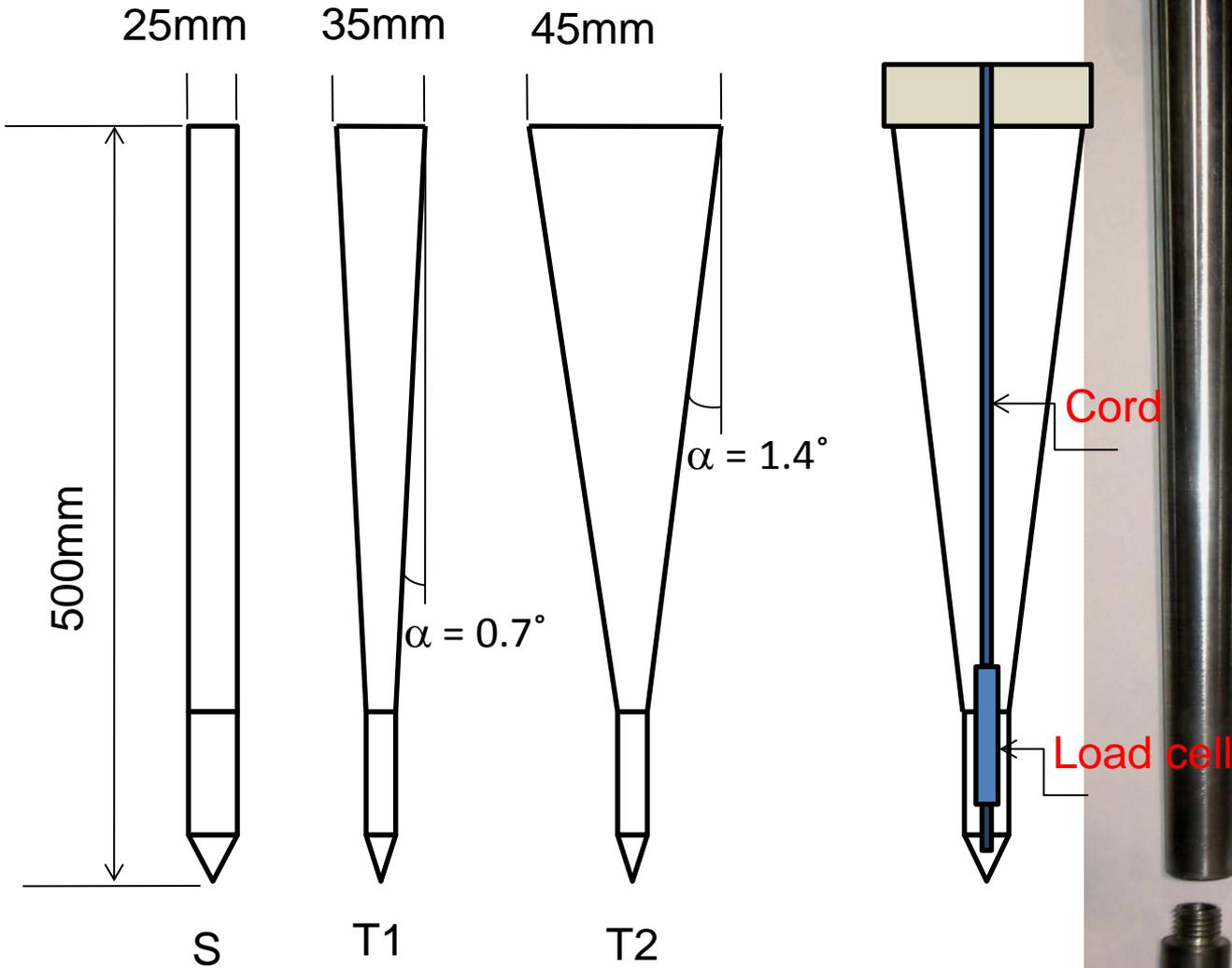
Gradation curve of sands



| Types of Model Piles | Naming | L mm | D _t mm | d mm | α ° | FRP reinforcement direction |
|----------------------------|------------------|------|-------------------|-------|------|-----------------------------|
| Smallest model steel piles | S' | 345 | 13 | 13 | 0.00 | |
| | T ₁ ' | 45 | 20 | 13 | 0.70 | |
| | T ₂ ' | 345 | 28 | 13 | 1.40 | |
| Smaller model steel piles | S | 500 | 25 | 25 | 0.00 | na |
| | T-1 | 500 | 35 | 25 | 0.70 | na |
| | T-2 | 500 | 45 | 25 | 1.40 | na |
| Prototype FRP piles | FC | 1524 | 168.3 | 168.3 | 0.00 | na |
| | T-3 | 1524 | 170.0 | 198.0 | 0.53 | 0° |
| | T-4 | 1524 | 159.0 | 197.0 | 0.71 | 0° |
| | T-5 | 1524 | 155.0 | 215.0 | 1.13 | 0° |

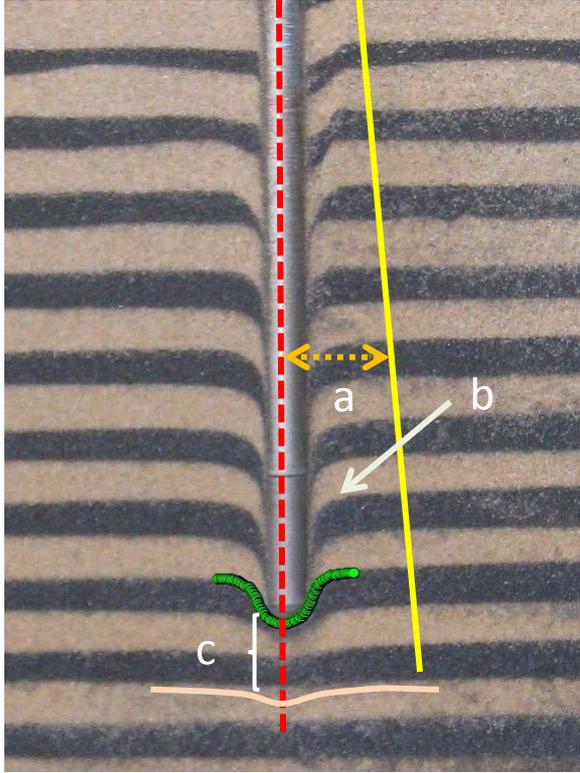
L: length of pile; D_t: diameter at the pile head; d: pile tip diameter; FRP: fiber-reinforced polymer; α: angle of tapering; na: not applicable

Steel Model Piles

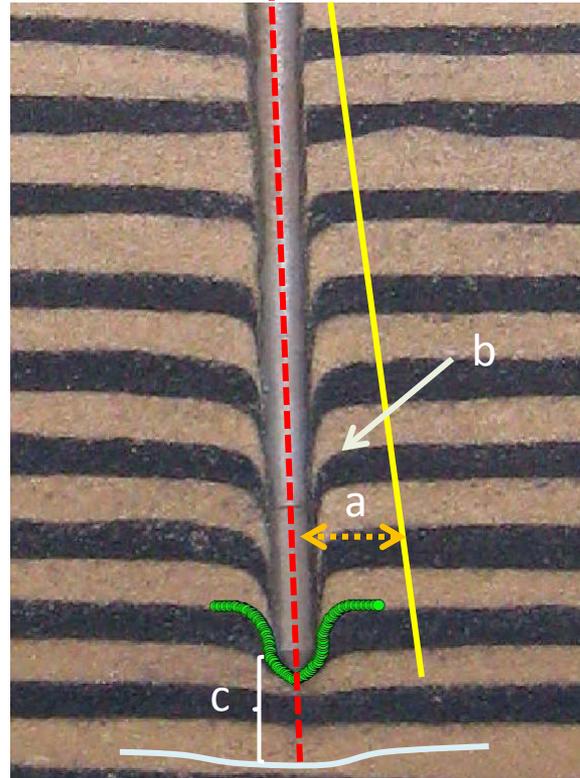




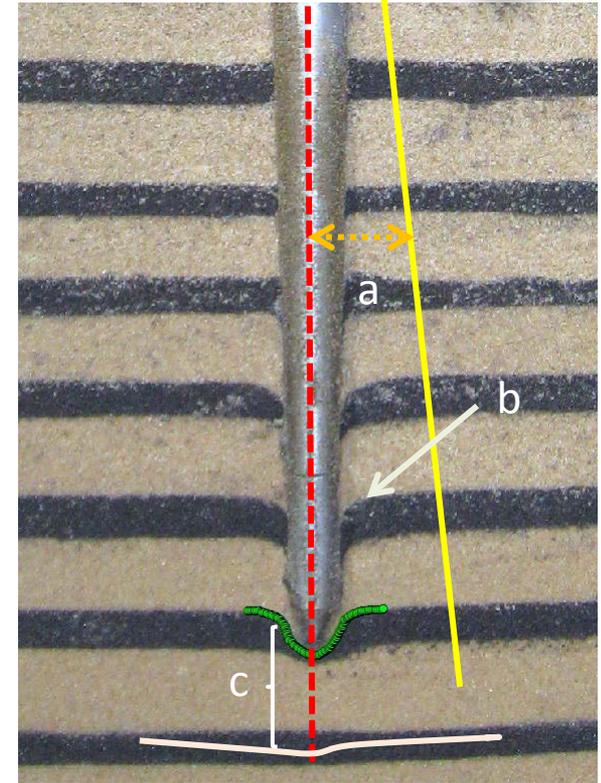
- Model test filled with **colored** TO ($I_D=0.8$) sand at *equal interval* and penetrated fully up to **10 cm**; then put in water bath to prevent from failure of model ground when chamber was split up.
- Trimmed soil carefully to observe mobilized mechanism.



$S' (\alpha = 0^\circ)$

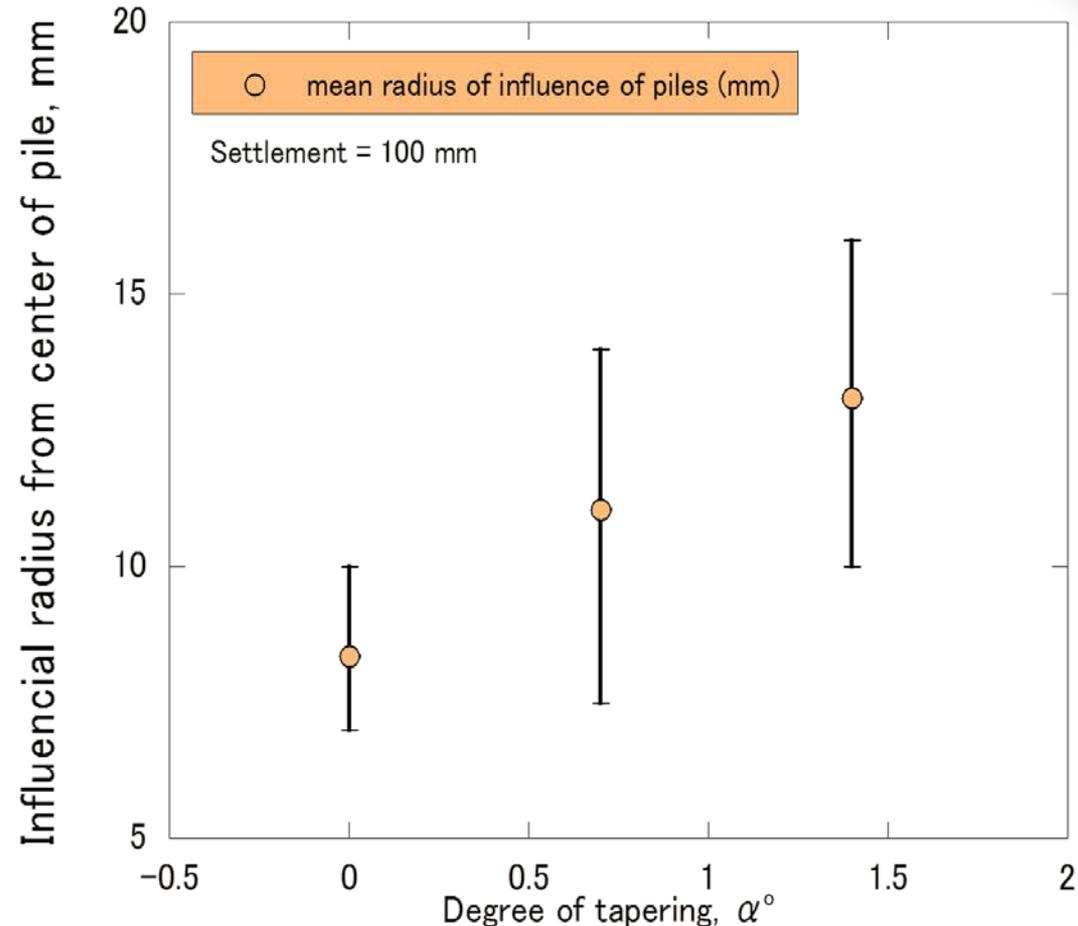


$T_1' (\alpha = 0.7^\circ)$



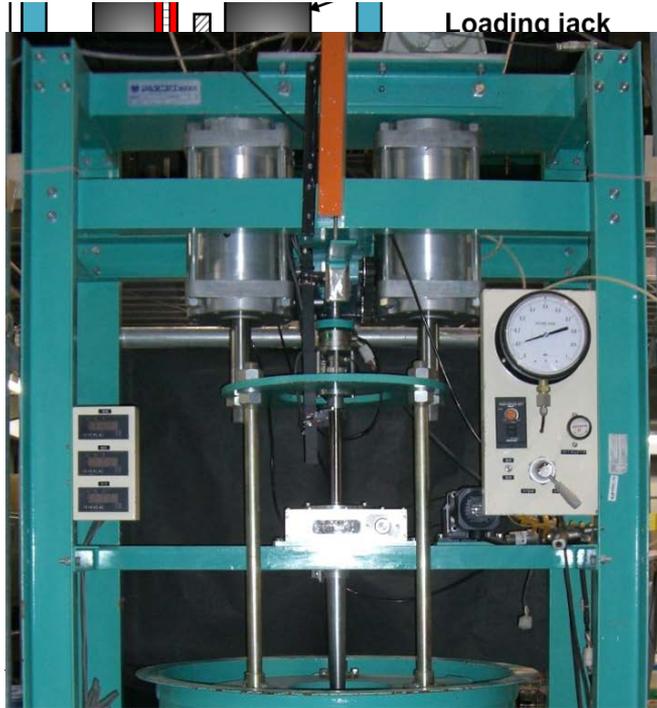
$T_2' (\alpha = 1.4^\circ)$

a = **effective radius of influence**, **increased** with tapering angle; b = **convex heave** due to effect of pile, **narrowed** with tapering angle; and c = **failure tip influenced zone**, **increased** with tapering angle



The most tapered pile showed the **highest radius of influence** that gives a strong evidence of *increases* in **skin friction** and **lateral stress** with minimizing the failure zone effectively.

Merits of Laboratory scale Pile Loading Tests

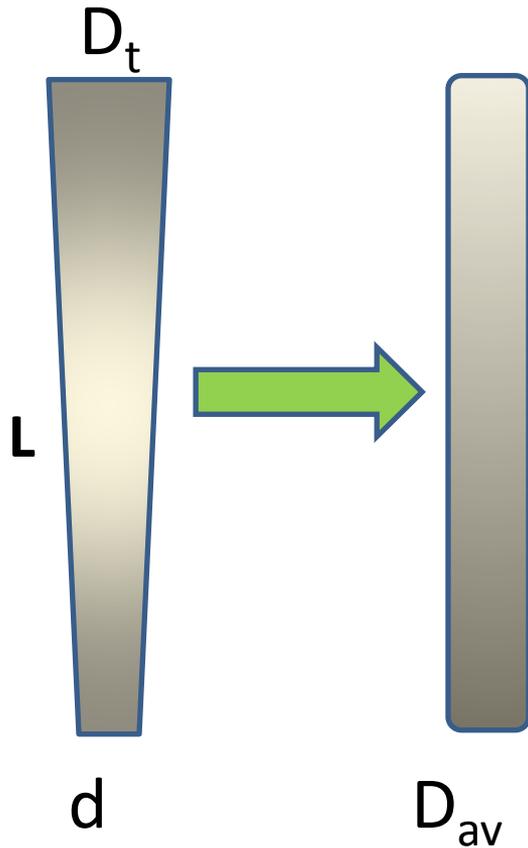


$$P_S = P_T - P_B$$

P_S : Total skin friction

P_T : Total load bearing

P_B : Total end bearing



$$\text{Unit skin friction, } f_s = \frac{P_S}{A_s}$$

$$D_{av} = \frac{(D_t + d)}{2}$$

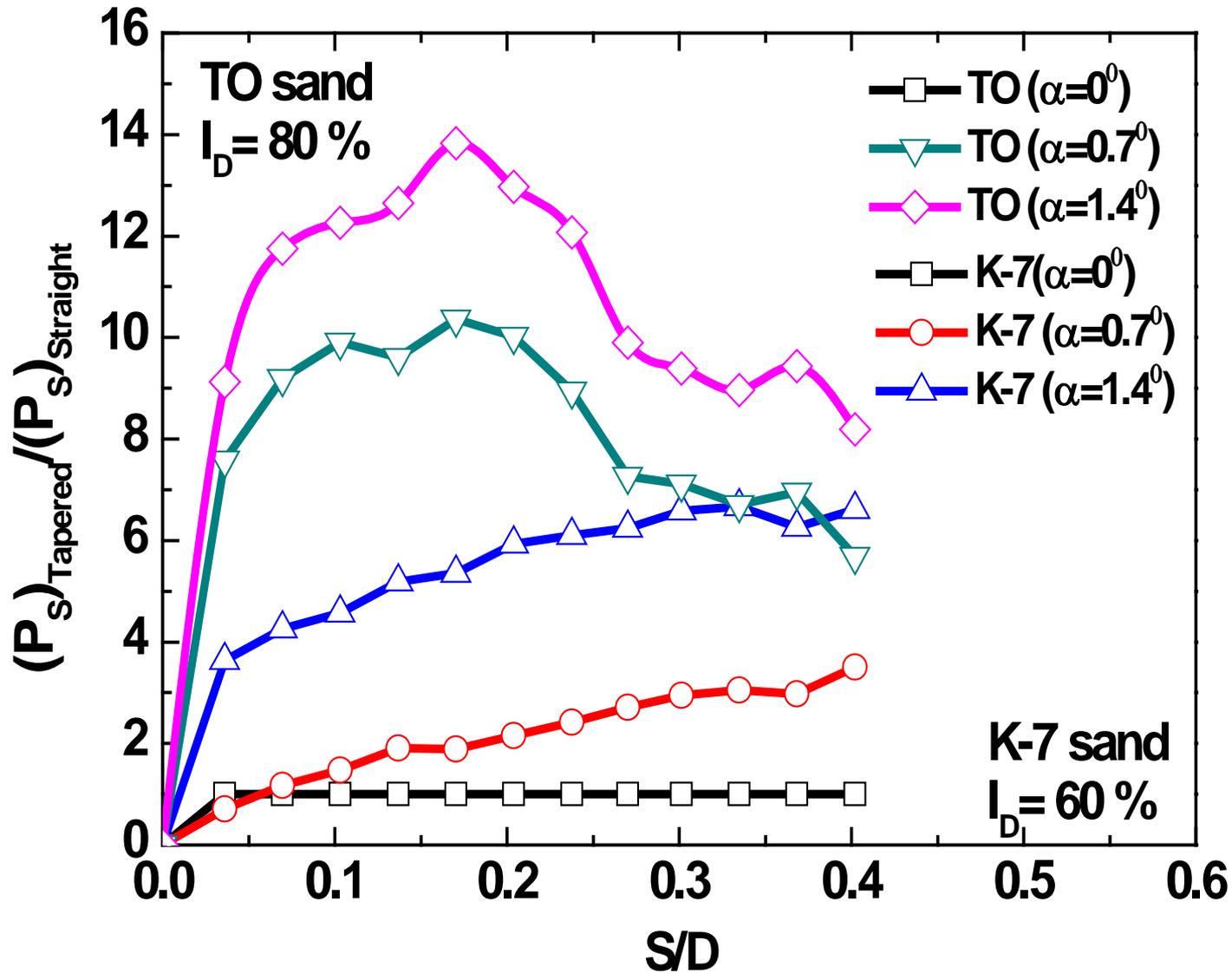
$$A_s = \pi D_{av} (L + \Delta L)$$

A_s : surface area

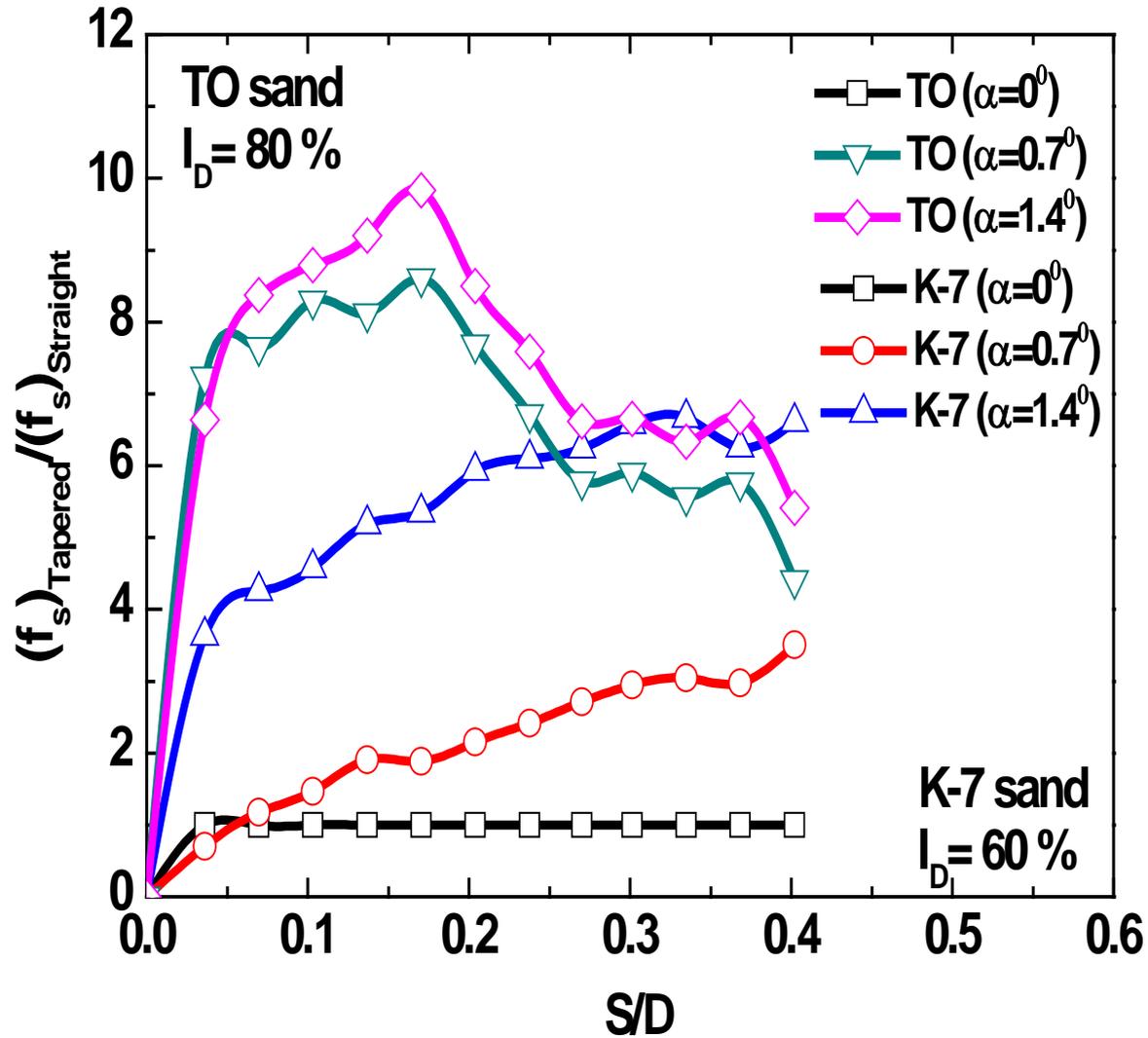
D_{av} : average diameter of pile head (D_t) and tip (d)

L : Effective length of pile

ΔL : Incremental depth of pile penetration



Skin friction increases with increasing tapering angle.



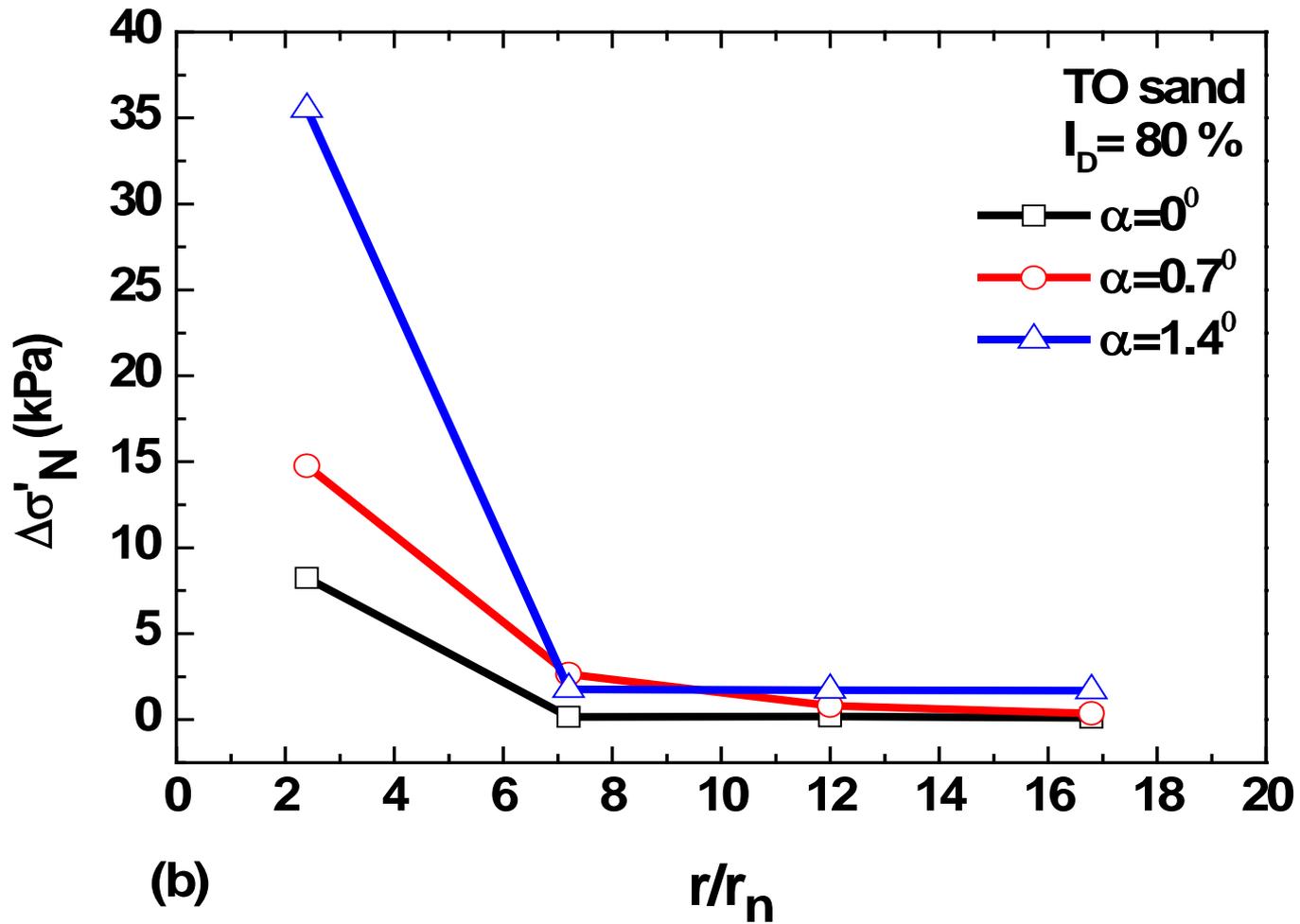
Unit skin friction also maintains the same trend.



Effects of lateral earth pressure



Radial distance from the center of the pile normalized by dividing distance of transducers to the pile tip radius (r/r_n)



Tapering effects are *higher* in the **most tapered piles** adjacent to pile-ground interface



Extended model for Evaluating Skin Friction



- The proposed model was extended after **Kodikara and Moore (1993)**; the model was incorporated for determination of **skin friction** using *cylindrical cavity expansion theory* by **Yu and Houlsby (1991)**.
- Generally, one of the soil parameters, either **angle of internal friction** or **dilatancy angle** is assumed to be **constant**.
- However, the **stress-dilatancy relationship** is interdependent on the **confining pressure**, **relative density** and **angle of internal friction**.

Concept of Determination of Skin Friction



After Kodikara & Moore (1993)

the **vertical pile movement** u_p *at any point X* on the pile-ground interface **>** **the vertical ground movement** u_g at the *corresponding point Y*. While the pile is displaced from point X to X', the ground moves from point Y to Y', obtaining the **lateral movement v**.

Pile ground slips but exhibits elastic deformation

$$\tau_x = \frac{K_e \tan \alpha \tan(\phi_i + \alpha) u_p + \sigma_0 \tan(\phi_i + \alpha) + c'_i}{1 + \frac{K_e r_m}{G} \tan \alpha \tan(\phi_i + \alpha)}$$

when $(u_p > (u_p)_Y)$ or $\sigma > \sigma_Y$, the plastic zone is developed *along with slippage* to obtain an elastic perfectly plastic pile-ground interface and plastic zones extends more. In this case, the radial stress (σ) will be changed into the form:

$$\sigma = \sigma_Y + \int_{v_Y}^v K_p dv$$

Where, v_Y can be computed using $(u_p)_Y$ and $(\tau_x)_Y$ which is the vertical shear stress when $u_p = (u_p)_Y$. Then, the corresponding vertical shear stress, τ_x can be expressed as:

$$\tau_x = \left(\sigma_Y + \int_{v_Y}^v K_p dv \right) \tan(\phi_i + \alpha) + c'_i$$

Stress-Dilatancy Relationship

- Generally, the dilatancy angle is considered to be zero for evaluating large strain analyses.
- But the real ground behaves the angle of internal friction and the rate of dilatancy at the critical state are as interdependent functions of density and effective stress.
- The density and confining pressure change significantly when a tapered pile penetrates with settlement ratios.
- The confining pressure increases with increasing relative density together with the angle of internal friction and dilatancy.
- Therefore the stress-dilatancy property is inserted in the cavity expansion theory (Yu and Houslby, 1991) and proposed model for determination of the skin friction by Kodikara and Moore (1993).

Stress-Dilatancy Relationship (Bolton, 1986,1987)



$$\phi'_{max} - \phi'_{cv} = 0.8 \psi_{max} = 5 I_R^0 \quad \text{and,} \quad I_R = I_D (10 - \ln p') - 1$$

- Where, ϕ'_{max} , ϕ'_{cv} , ψ_{max} , and I_R^0 are the maximum angle of friction, the angle of friction at critical states, maximum dilation angle and the relative dilatancy index at plane strain.
- The relative dilatancy index I_R is a function of relative density I_D and mean effective stress p' .
- A plastic zone will be obtained at the cavity wall within the region $a \leq r \leq b$, with an increment of cavity pressure p . By partitioning elastic and plastic regions, the stress component at the plastic region that satisfies the equilibrium condition as:

$$p' = -p_0 b \frac{(\alpha' - 1)}{\alpha'} r^{-\frac{(\alpha' - 1)}{\alpha'}}$$

At the boundary of plastic region where $r \leq a$, the effective mean stress can be modified to:

$$p' = -p_0 R$$



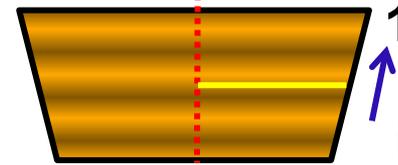
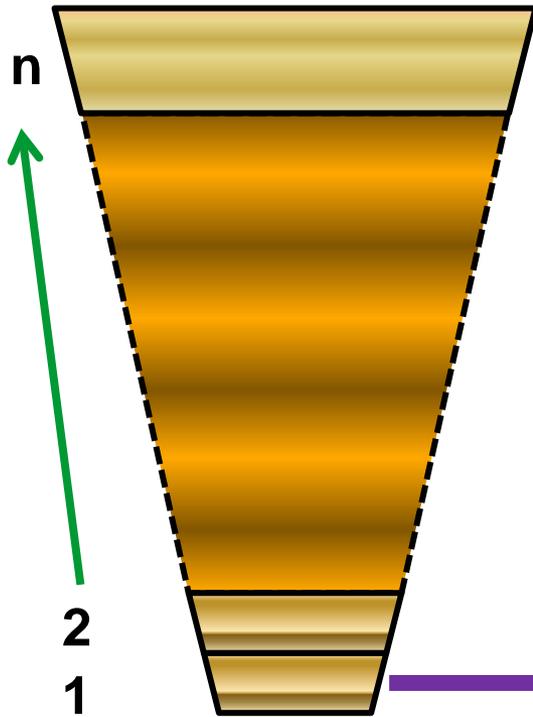


Results of the Model using Load Transfer Method



- The **load transfer method** proposed by **Coyle and Reese (1966)** [based on **Seed and Reese (1957)**] is used to estimate the **skin friction** by inserting a **stress-dilatancy property** as the **extended model**.
- A **small settlement** at the **pile base** is **specified** and the **axial load** at the **top of** this **segment** is **iteratively synchronized** to **satisfy** the **equilibrium condition**, and the **process** undergoes to **next segment** to **calculate** the **settlement**.

Proposed Extended Model



Compute ϕ and ψ using σ_i

Assume ϕ' : small

Compute $\phi'_{\text{new}} = 5I_R^\circ$ using σ_i

No

$\phi'_{\text{new}} = \phi'$

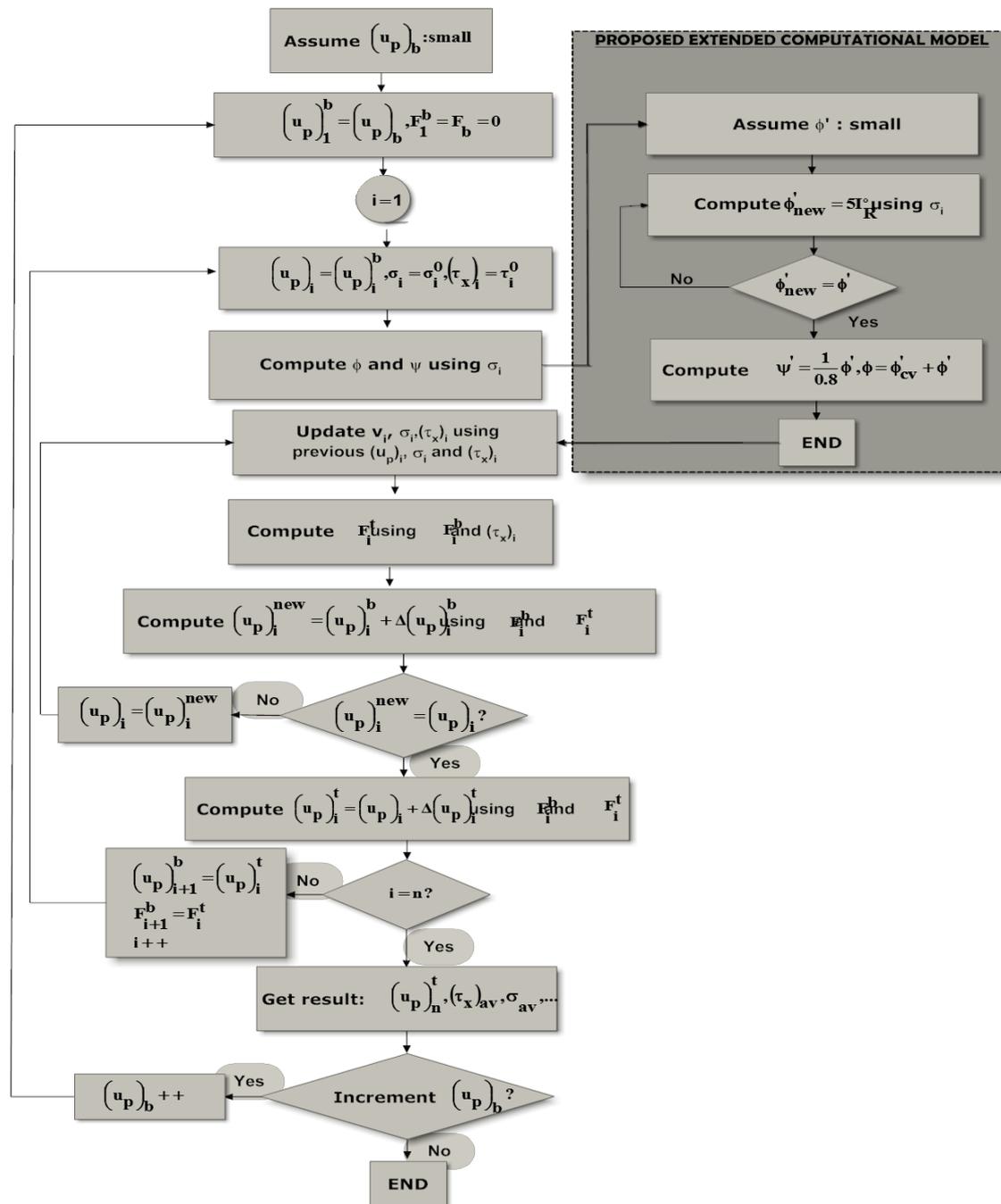
Yes

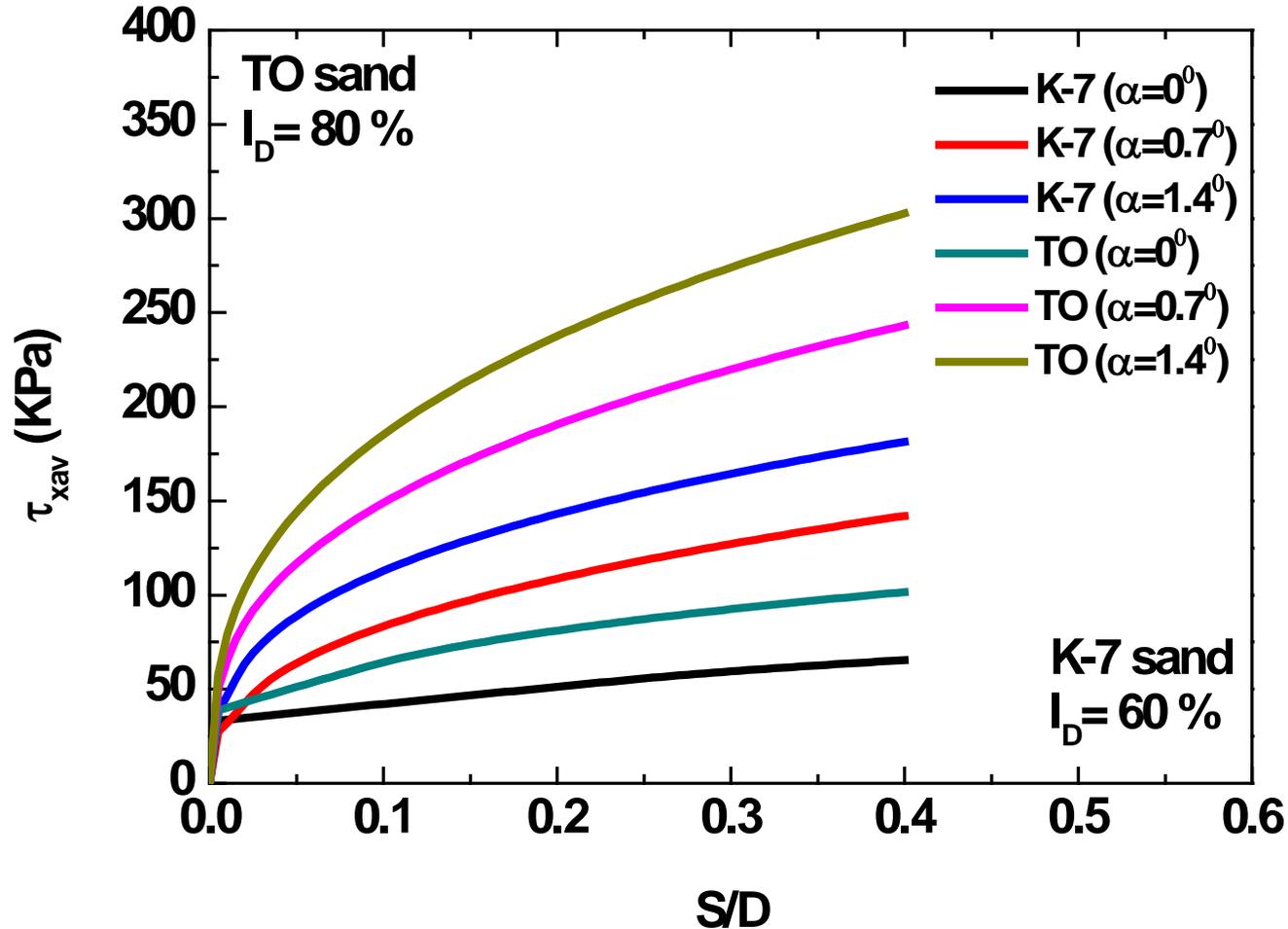
Compute $\psi' = \frac{1}{0.8}\phi'$, $\phi = \phi'_{\text{cv}} + \phi'$

Assume $(u_p)_b$: small

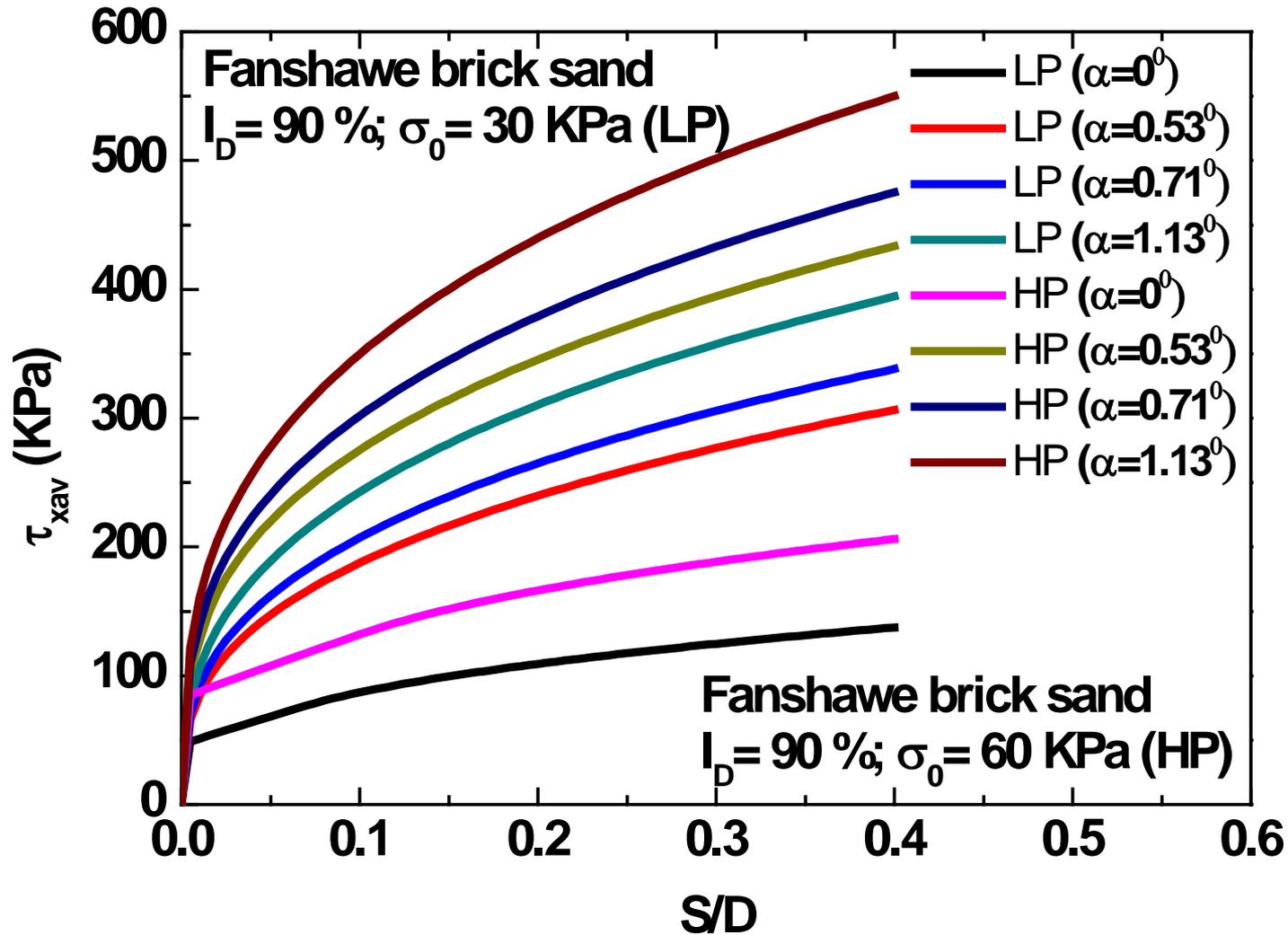
$$\begin{pmatrix} u_p \end{pmatrix}_1^b = \begin{pmatrix} u_p \end{pmatrix}_b, F_1^b = F_b = 0$$



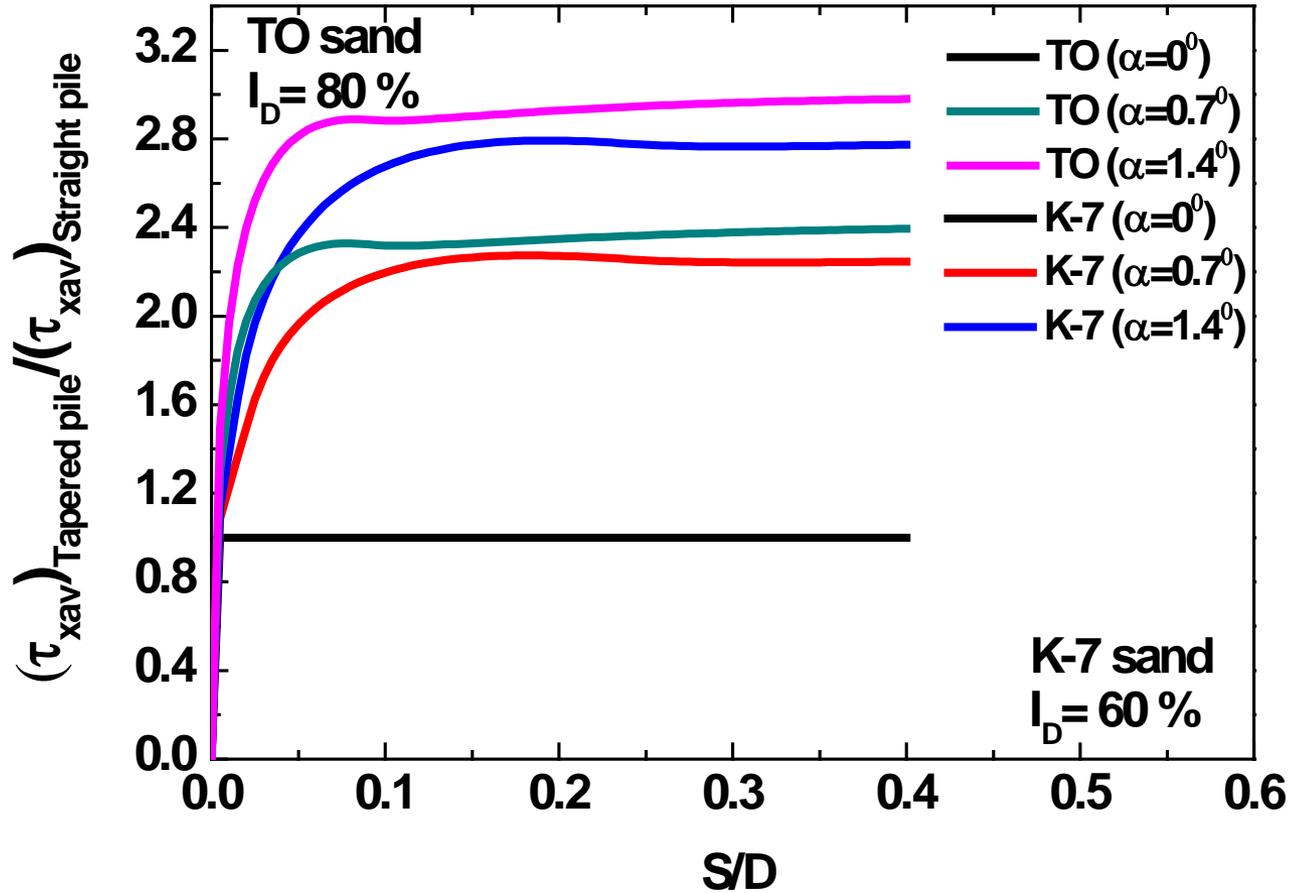




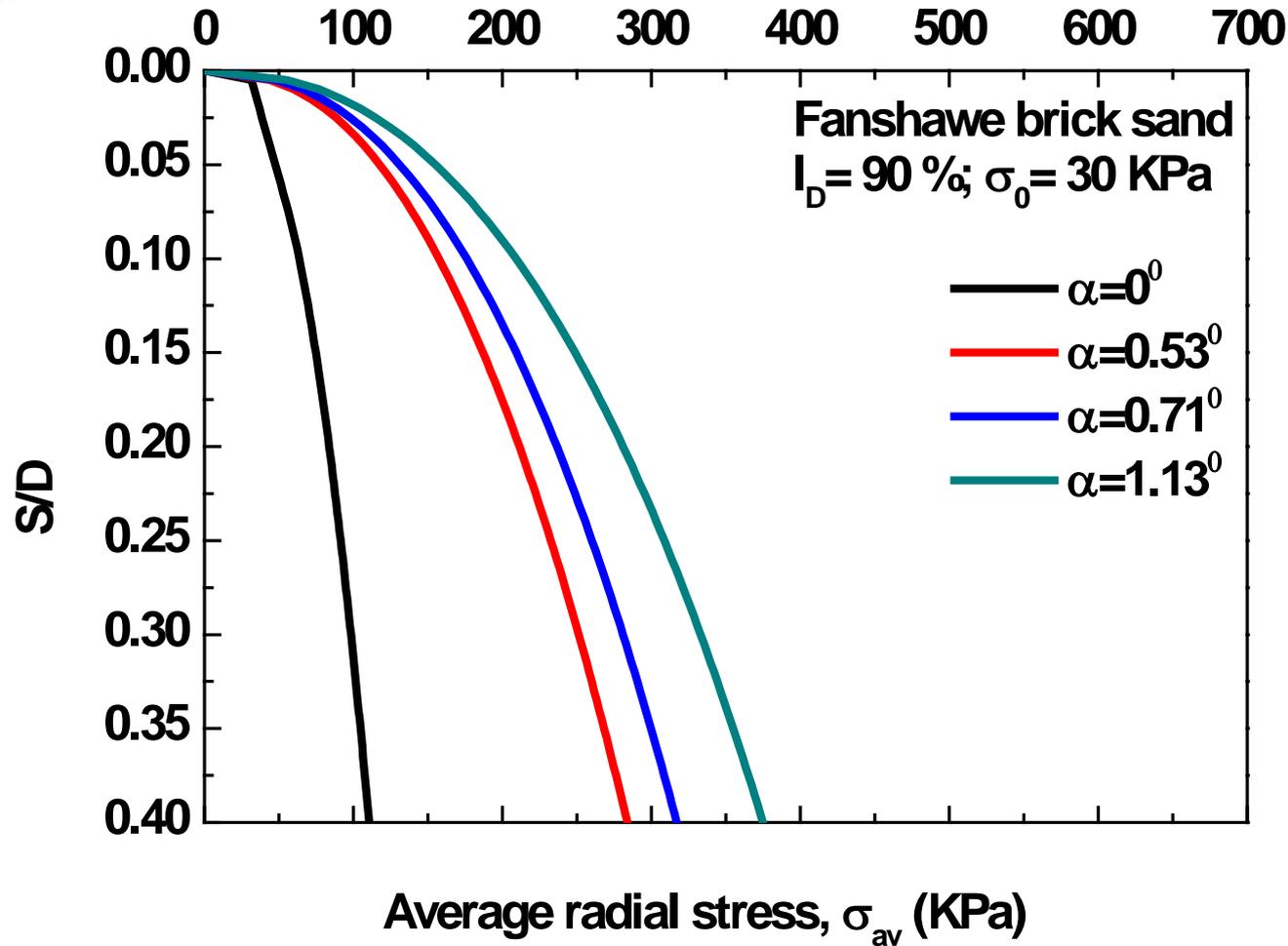
The vertical shear stress **increases** with increasing **tapering angles** for different types of sandy ground.



For **different ground** and **confining pressures**, the **skin friction increases** with the **same ratio** irrespective of soil type



Normalized shear stress of the most tapered pile shows remarkable improvement on skin friction.



Average radial stress of *the most tapered pile* increased remarkably with settlement ratios.

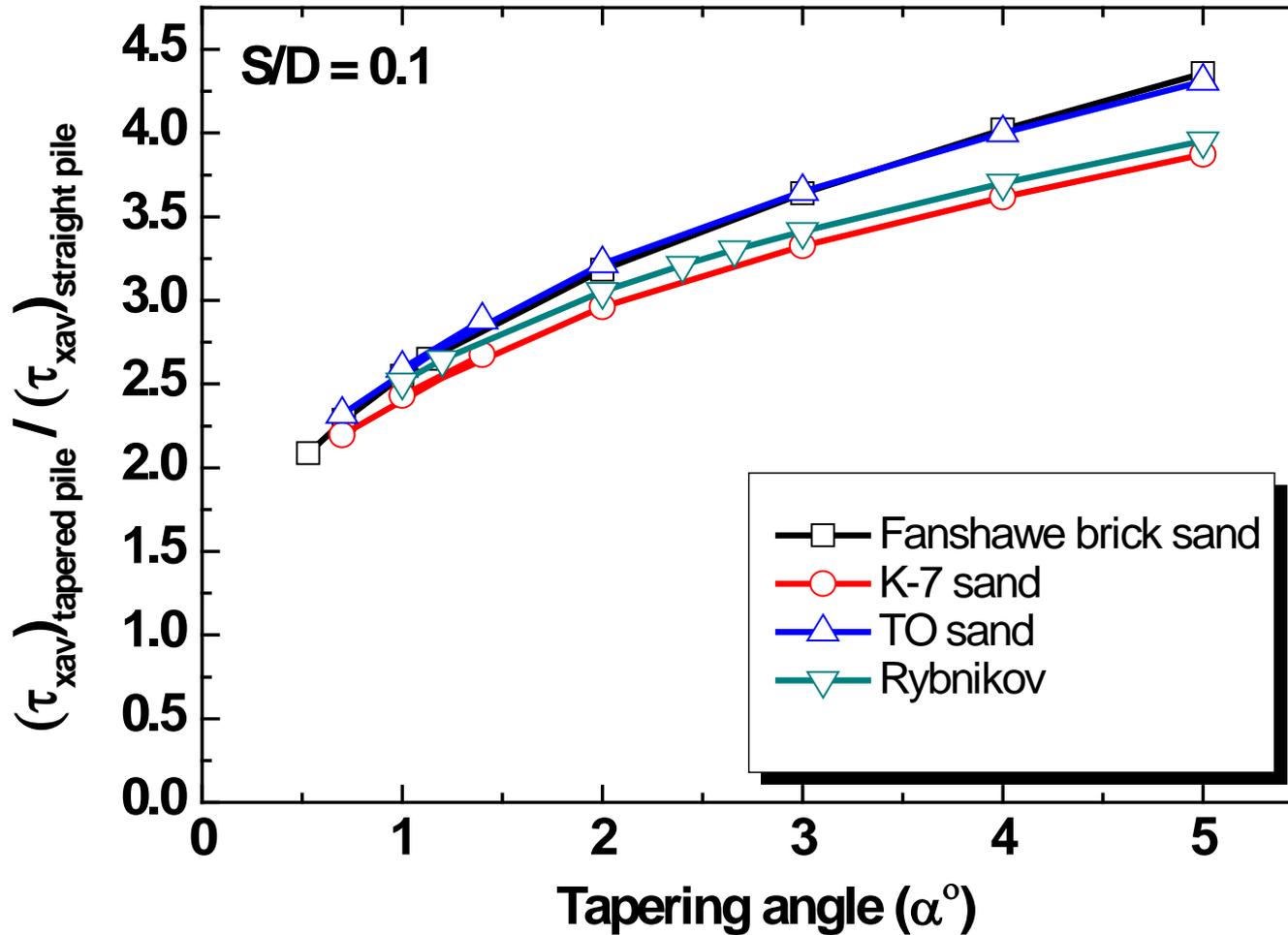


Parametric Study and Validity of the Model

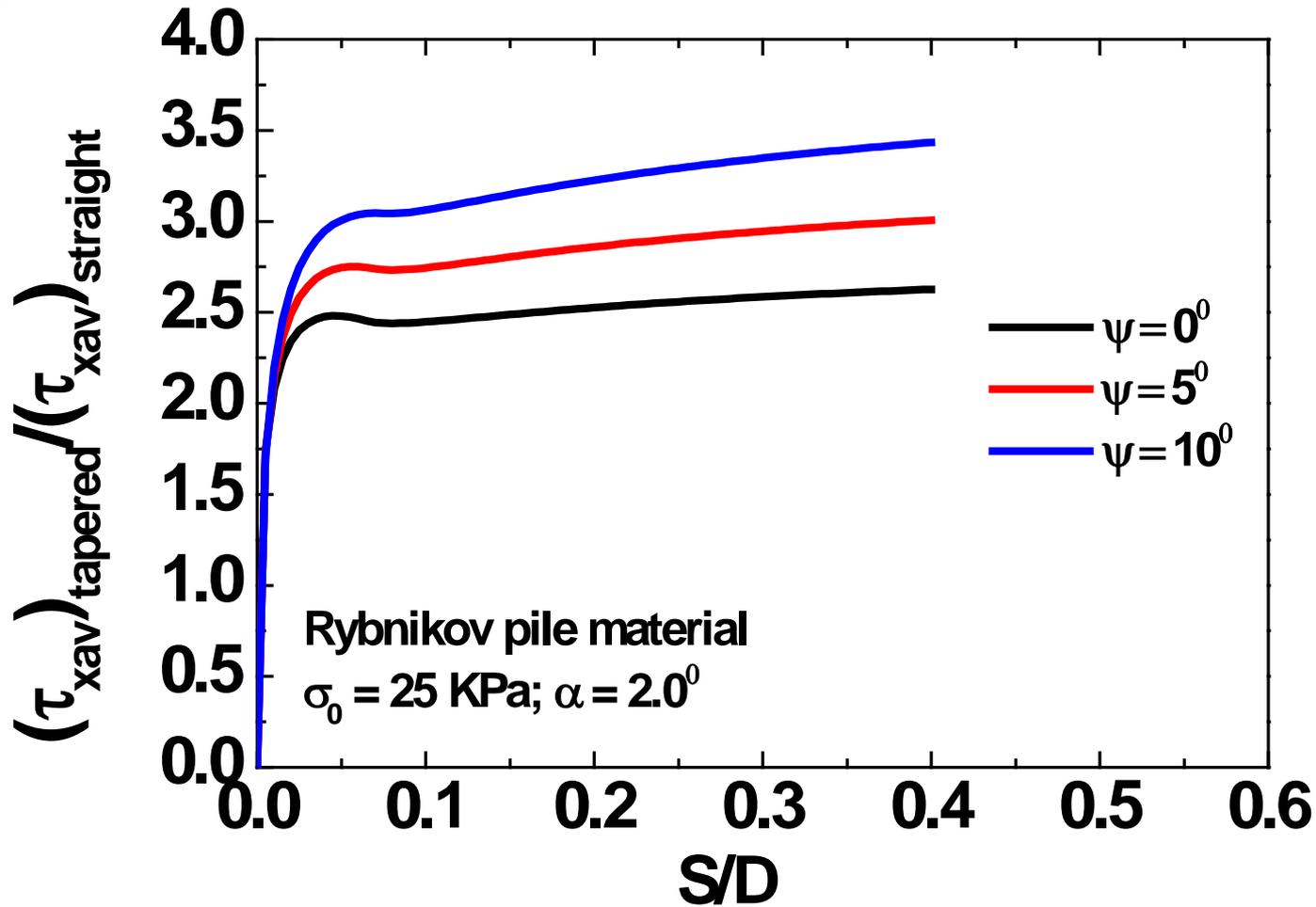


A real type **Rybnikov (1990) pile** and **prototype pile (Sakr et. al, 2004, 2005, and 2007)** are accomplished to check and validate the applicability of the proposed model.

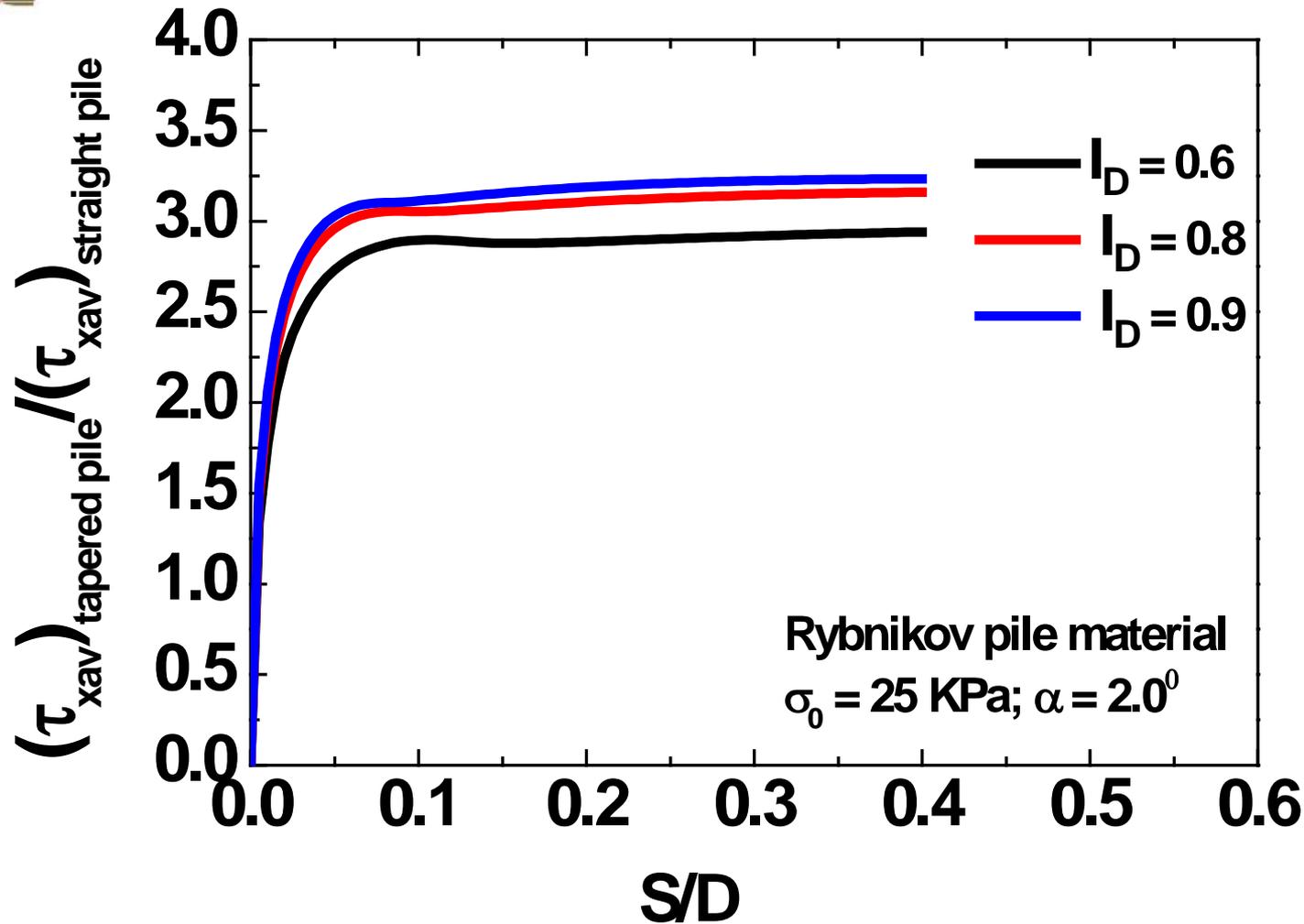
| Parameters | Rybnikov Pile Material | | | | | | | | |
|------------------|--------------------------|--------------|----------------|----------------|-----------------|--------------|--------------|--------------|--------------|
| | Default ($\alpha = 2$) | $\alpha = 0$ | $\alpha = 1.2$ | $\alpha = 2.4$ | $\alpha = 2.66$ | $\alpha = 1$ | $\alpha = 3$ | $\alpha = 4$ | $\alpha = 5$ |
| G, MPa | Formula | Formula | Formula | Formula | Formula | Formula | Formula | Formula | Formula |
| C, KPa | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| c_v , KPa | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| ϕ° | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration |
| ψ° | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration | Iteration |
| L, mm | 4500 | 4500 | 4500 | 4500 | 4500 | 4500 | 4500 | 4500 | 4500 |
| α° | 2 | 0 | 1.2 | 2.4 | 2.66 | 1 | 3 | 4 | 5 |
| I_D | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| σ_o , KPa | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| ϕ'_{cv} | 32° | 32° | 32° | 32° | 32° | 32° | 32° | 32° | 32° |



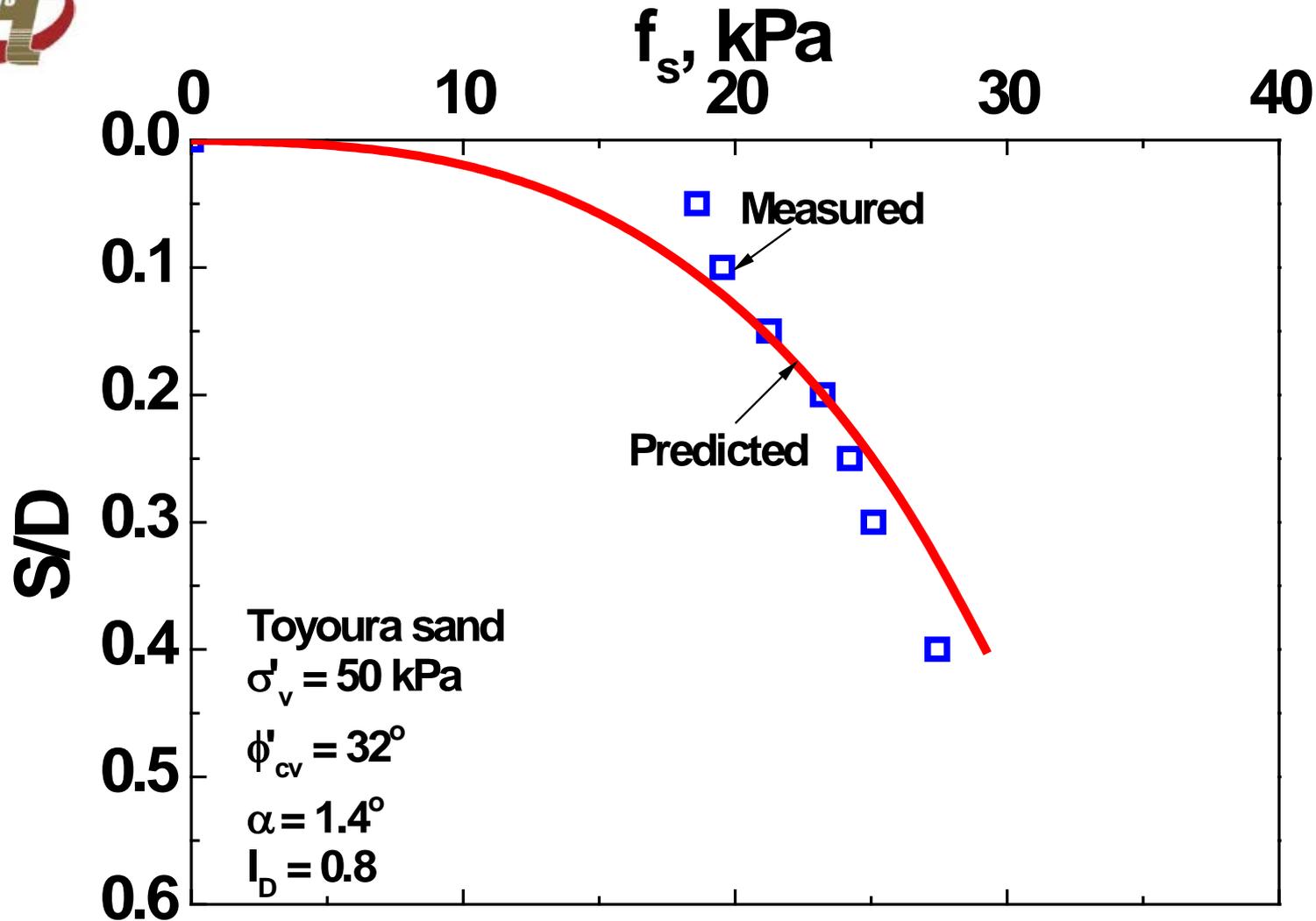
Skin friction increased remarkably together with *increasing tapering angle* at settlement ratio of 0.1.



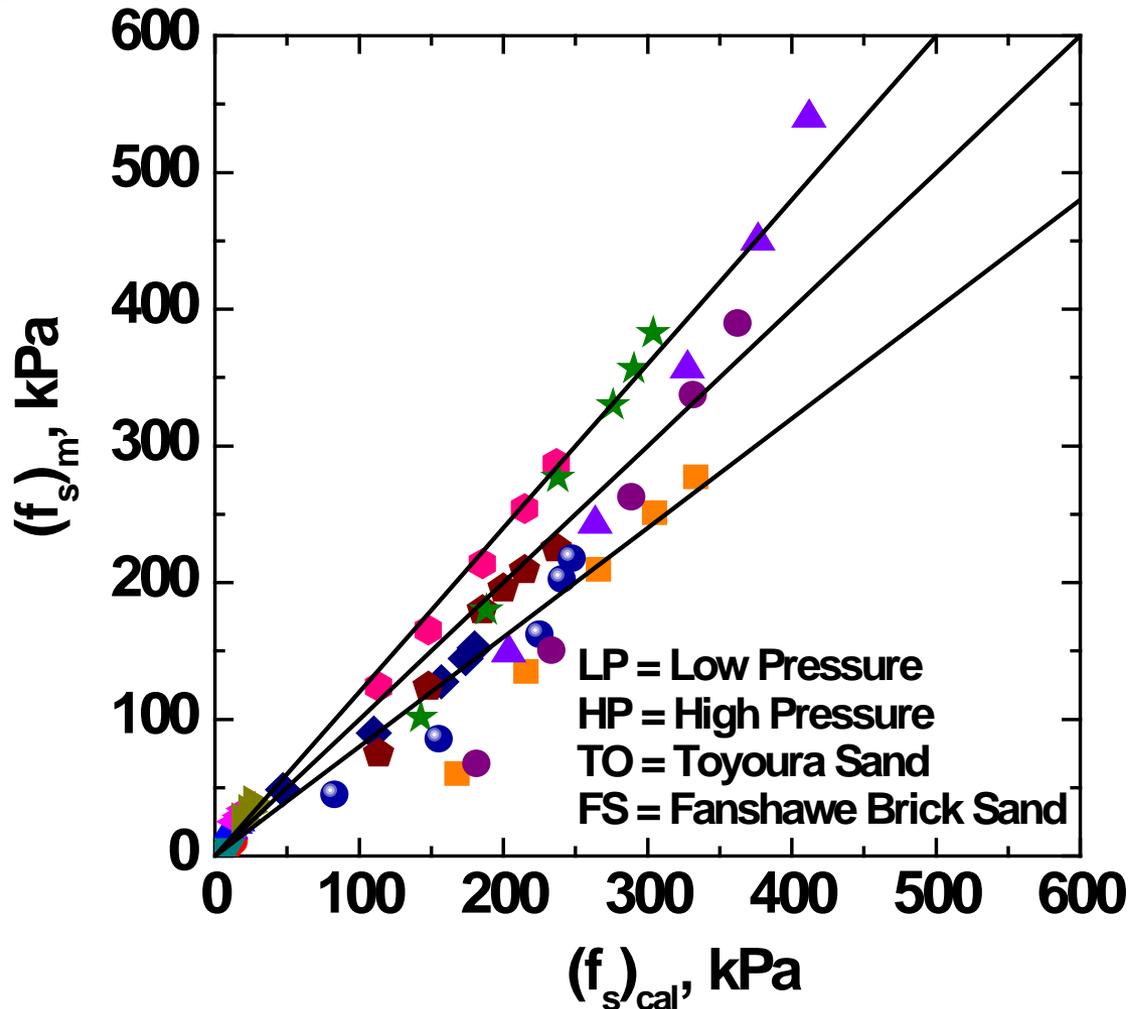
Normalized average vertical shear stress increased with *increasing dilatancy angles*.



Normalized average vertical shear stress increased with *increasing relative densities* of the ground.



The **measured** and **predicted** *skin frictions* lie near to each other with increasing settlement ratios.



- Legend**
- K-7, $\alpha=0^\circ$
 - K-7, $\alpha=0.7^\circ$
 - ▲ K-7, $\alpha=1.4^\circ$
 - ▼ TO, $\alpha=0^\circ$
 - ◀ TO, $\alpha=0.7^\circ$
 - ▶ TO, $\alpha=1.4^\circ$
 - ◆ FS, $\alpha=0^\circ$ (LP)
 - ◆ FS, $\alpha=0.53^\circ$ (LP)
 - ◆ FS, $\alpha=0.71^\circ$ (LP)
 - ★ FS, $\alpha=1.13^\circ$ (LP)
 - FS, $\alpha=0^\circ$ (HP)
 - FS, $\alpha=0.53^\circ$ (HP)
 - FS, $\alpha=0.71^\circ$ (HP)
 - ▲ FS, $\alpha=1.13^\circ$ (HP)

The measured and predicted *unit skin frictions* validated for different types of *piles* and *sandy ground*.



Conclusions

- The **mobilized mechanism** of **skin friction** shows that the **effective radius of the influenced zone** around the pile shaft **increases** in line with increases in the **tapering angle**.
- The **extended model** with the inserted **stress-dilatancy property** can **predict skin friction** using **cylindrical cavity expansion theory** in closed form solution and can easily determine with **simple fundamental properties of soils**.
- The **predicted skin friction** using the extended model shows **good agreement** with **measured** skin friction from various sources.

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**Thank you for
your kind attention !!!**