Analyses of Lateral Loaded Piles with P-Y Curves - Observations on the Effect of Pile Flexural Stiffness and Cyclic Loading

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Session 3B - Geotechnical II (Room: Pinehurst) - Paper: 3B-1_A49

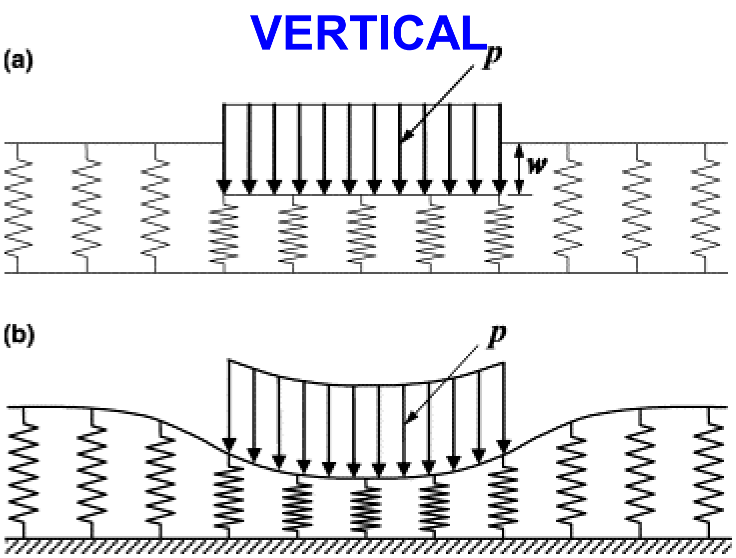
Outline

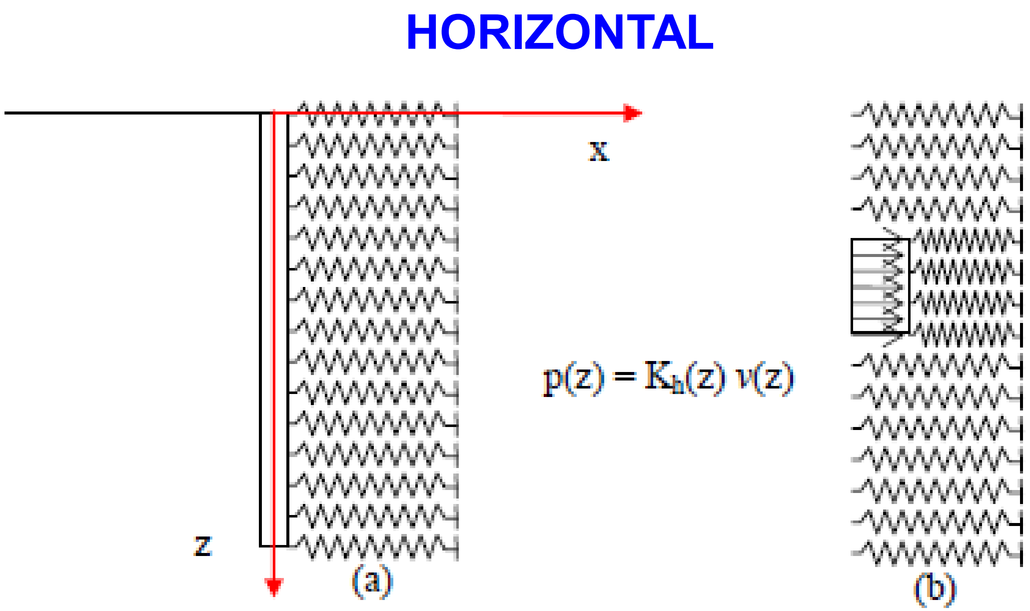
- Background
  - Beam on Elastic Foundation (BEF)
  - Subgrade Reaction
  - Beam Theory (related to net soil reaction per unit length (**P**) and pile lateral deflection (**Y**)
- P-Y curves
  - Elements of a P-Y curve (preferred terminology)
  - Commonly used P-Y Curves (empirical basis)
- The analytical methodology
- Possible limitations/challenges
  - Influence of pile cross section and EI?
  - Effects of lateral load cycles?
- Summary and Conclusions

BACKGROUND

Beam on Elastic Foundation (BEF)

- After Winkler (1867) aka as Beam on Winkler Foundation (BWF)



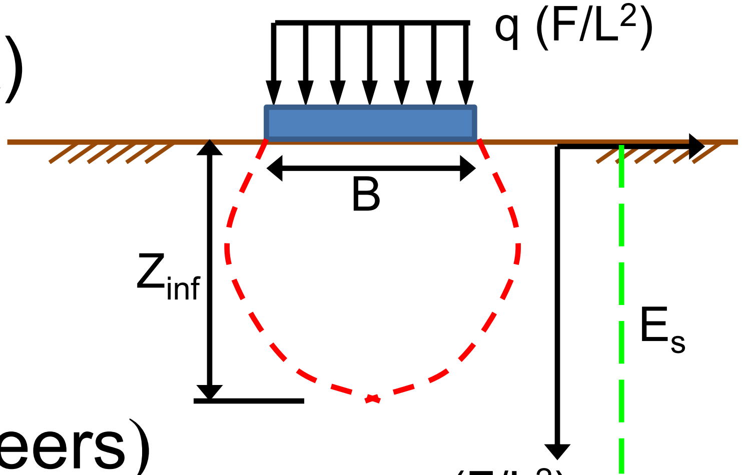


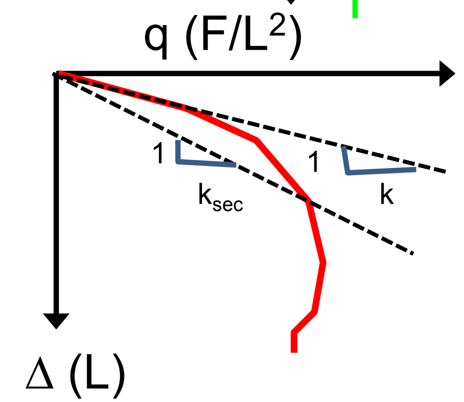
k_h = Modulus of Lateral (Horizontal) Subgrade Reaction [F/L^3]

Typically linear representation of soil reaction.

Vertical Modulus of Subgrade Reaction

- Modulus of subgrade reaction (k)
- Is it a soil property? **NO**
- Footing, rafts, pavement design
- Westergaard's work in 1920's
- $q = k\,\Delta$ (used by structural engineers)
  - q = applied or contact pressure [F/L^2]
  - Δ = settlement of footing under q [L]
- $k = q/\Delta$ = slope (linear spring constant) = modulus of subgrade reaction [F/L^3]

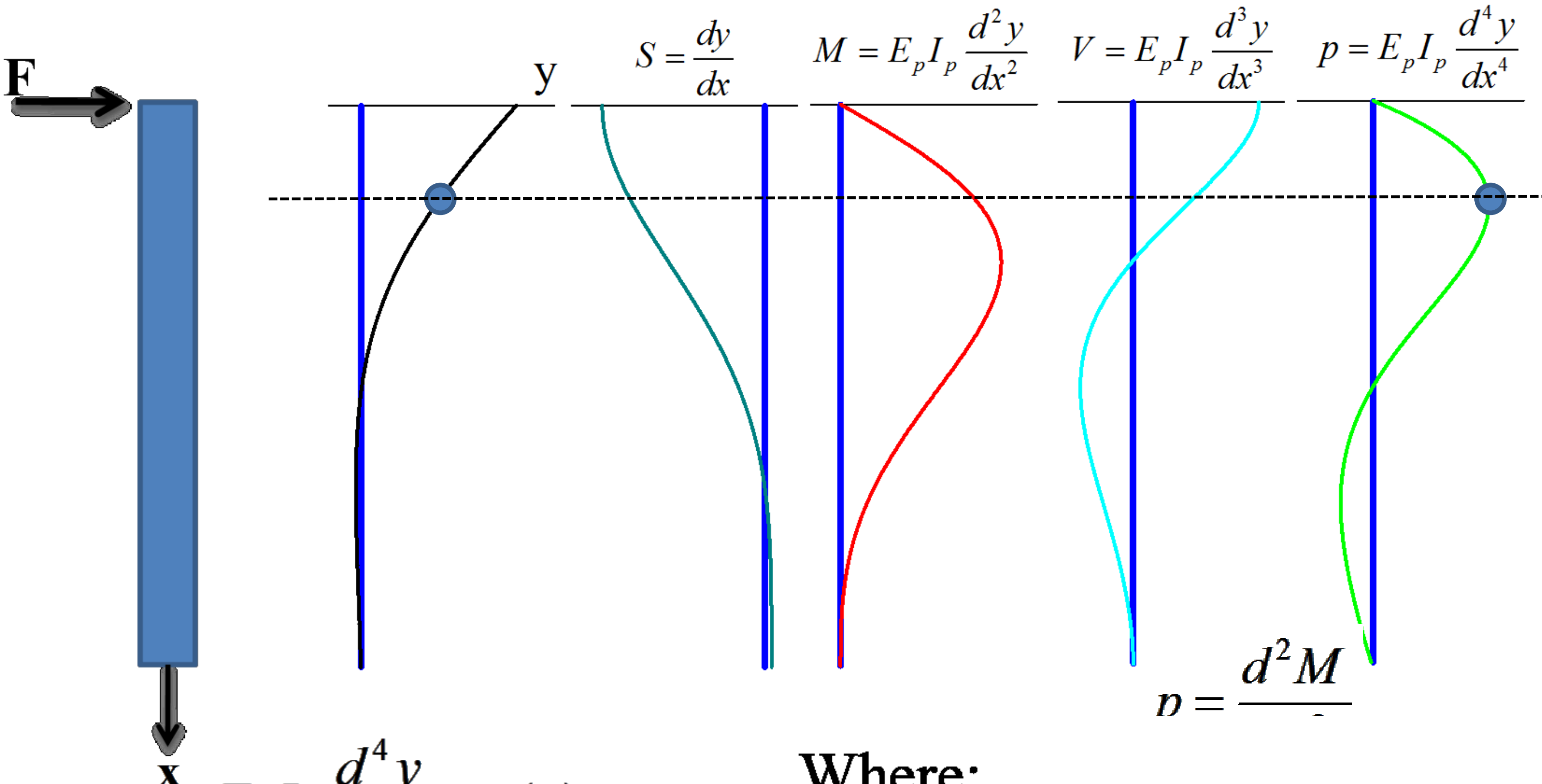




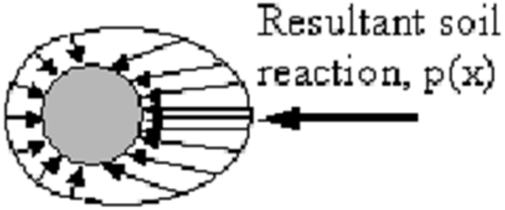
Vertical Modulus of Subgrade Reaction (Continued)

- Elastic settlement of circular footing:
- $\Delta = I\,\frac{q.B.(1-\mu^2)}{E_s}$
- μ = 0.5 (undrained) ~ 0.3 (drained)
- $E_s = E_u$ (undrained) vs E' (drained)
- $k_v = {}^{q}/_{\Delta} = \frac{E_s}{I.B.(1-\mu^2)}$
- **Not a fundamental soil property**
- Not readily measured
- Depends on many factors such as size and shape of footing, type of soil, relative stiffness of footing and soil, vary along footing, vary with time, etc.

Beam theory



$$E_pI_p \frac{d^4y}{dx^4} - p(x) = 0$$



Where:

E_pI_p = Flexural stiffness of pile;

y = Lateral deflection;

x= Depth below the pile head ;

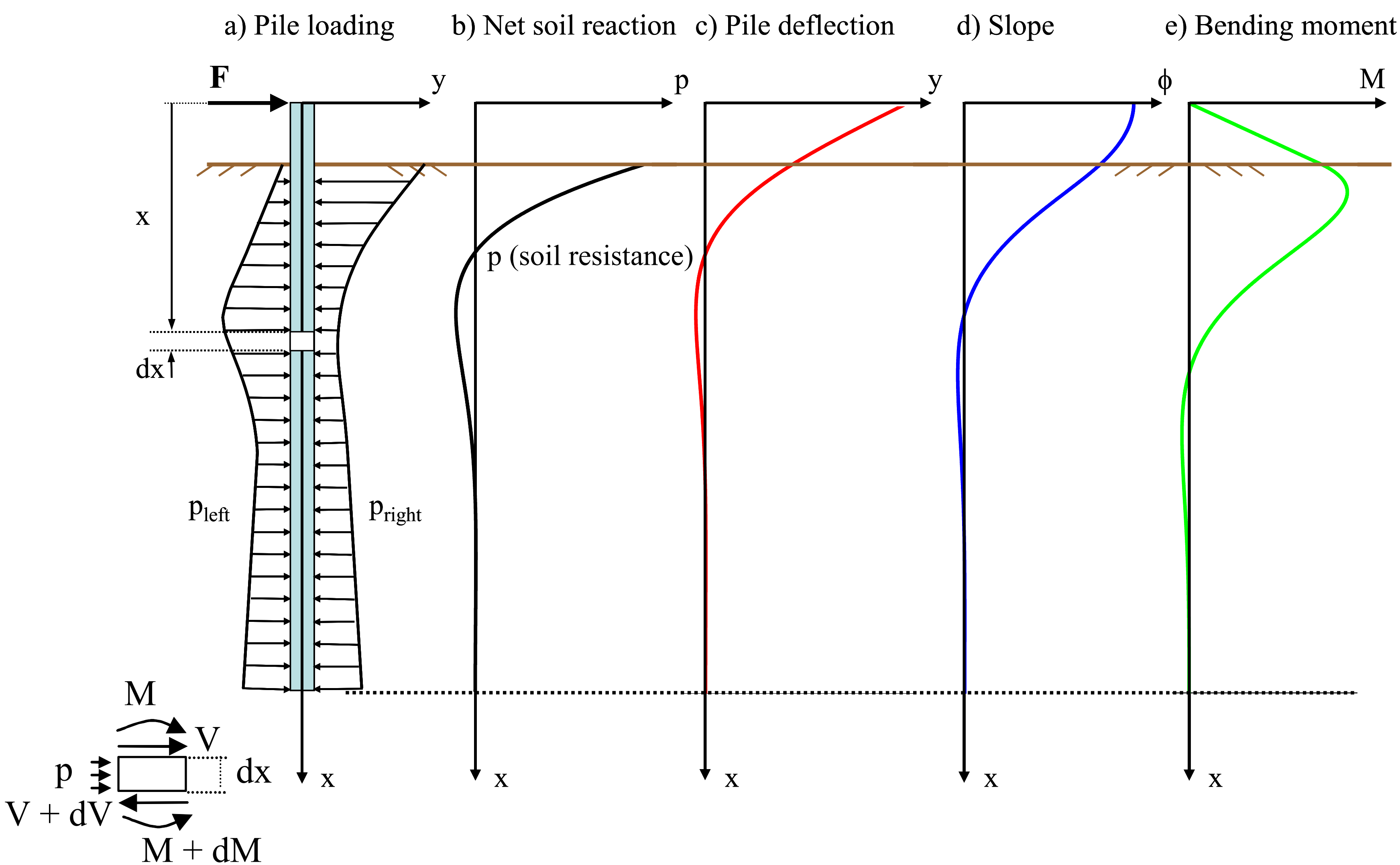
p= Soil reaction per unit length of pile.

Table 1 Relationships commonly used for elastic piles in flexion

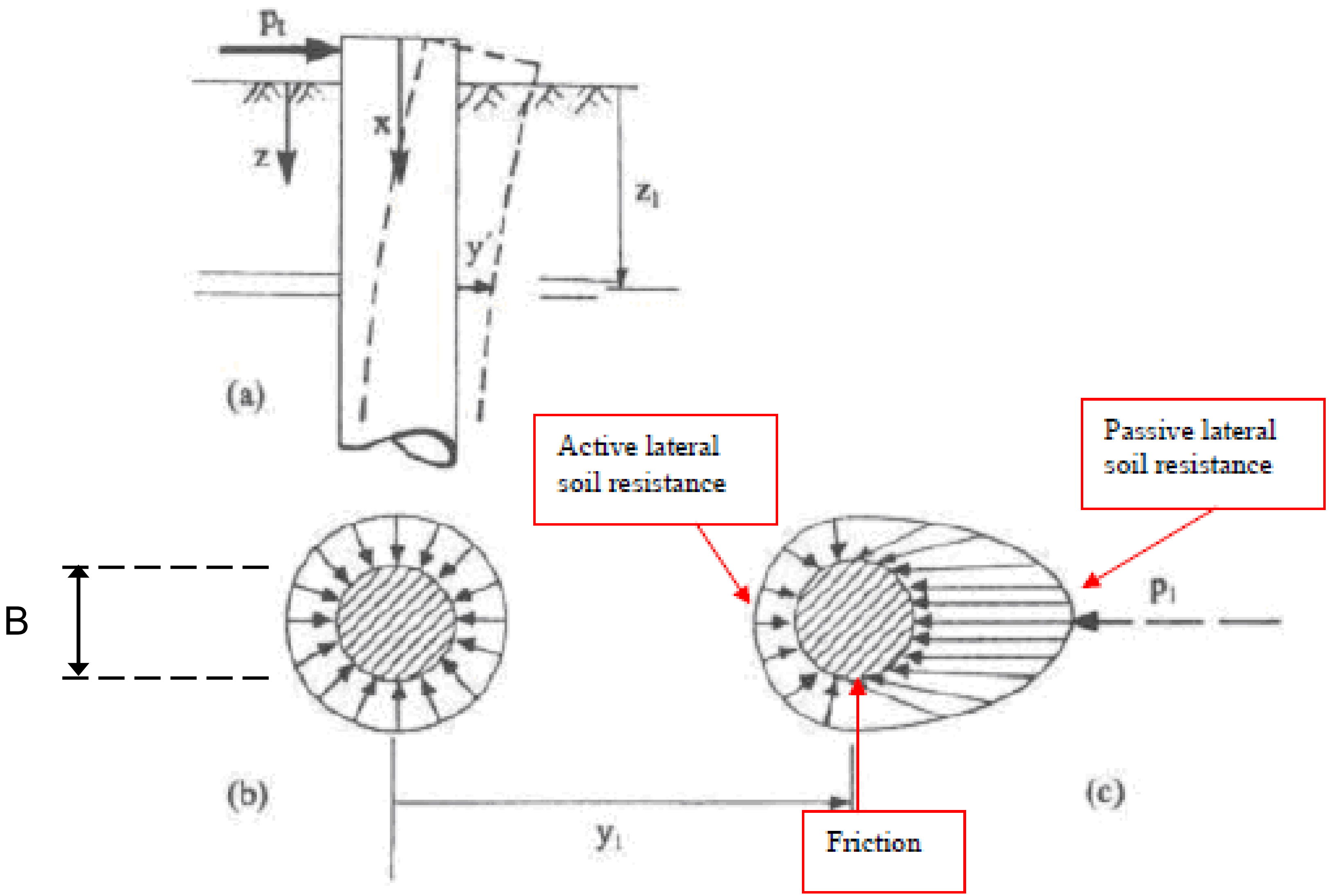
Variable	Formula	Units
Distance along the length of the pile (measured from pile head)	x	[L]
Distance to neutral axis within pile cross section	z	[L]
Deflection	y	[L]
Slope or rotation of pile section	$\phi = \frac{dy}{dx}$	[Dimensionless]
Curvature	$\kappa = \frac{d^2y}{dx^2}$	[Radians/L]
Bending moment	$M = E_pI_p \cdot \frac{d^2y}{dx^2} = E_pI_p \cdot \kappa$	[F x L]
Shear force	$V = E_pI_p \cdot \frac{d^3y}{dx^3}$	[F]
Axial load	Q	[F]
Soil reaction (or load intensity)	$p = E_pI_p \cdot \frac{d^4y}{dx^4}$	[F/L]

Notes: E_pI_p = flexural stiffness of pile, where E_p = elastic modulus of pile material, and I_p = moment of inertia of pile cross section with respect to the neutral axis

Relationships between variables

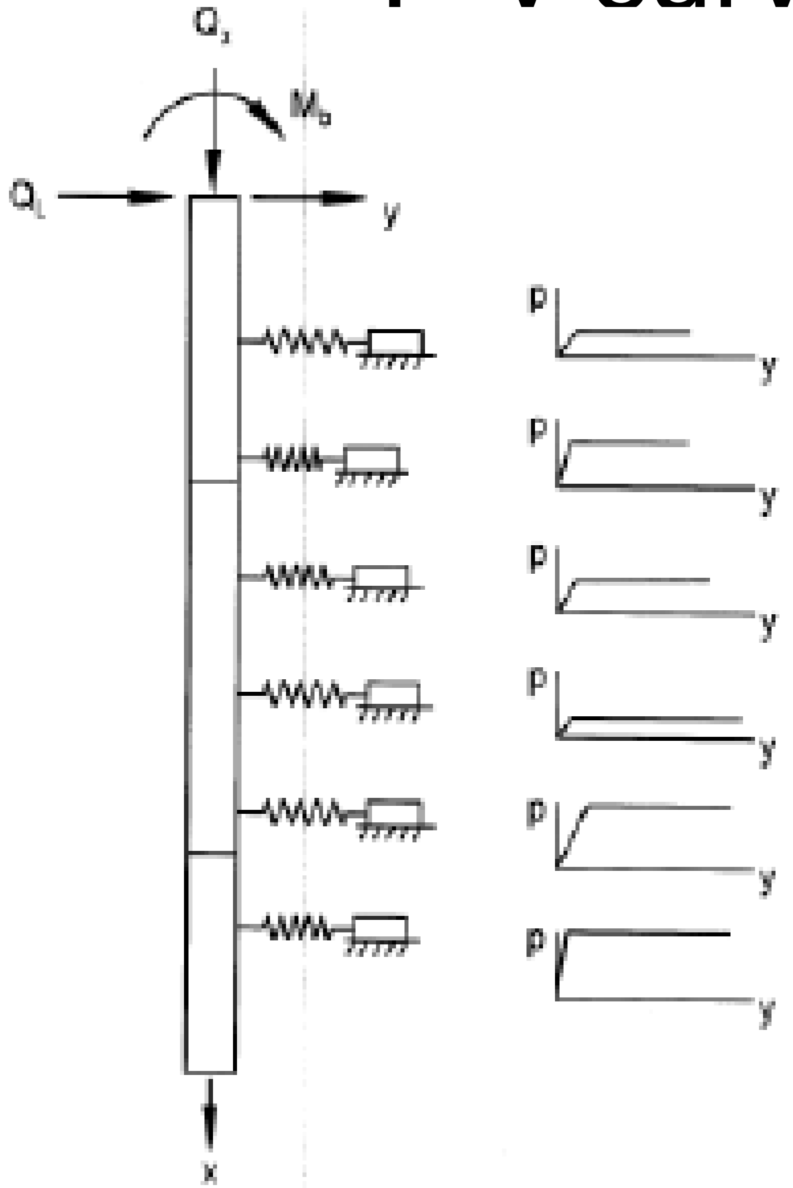


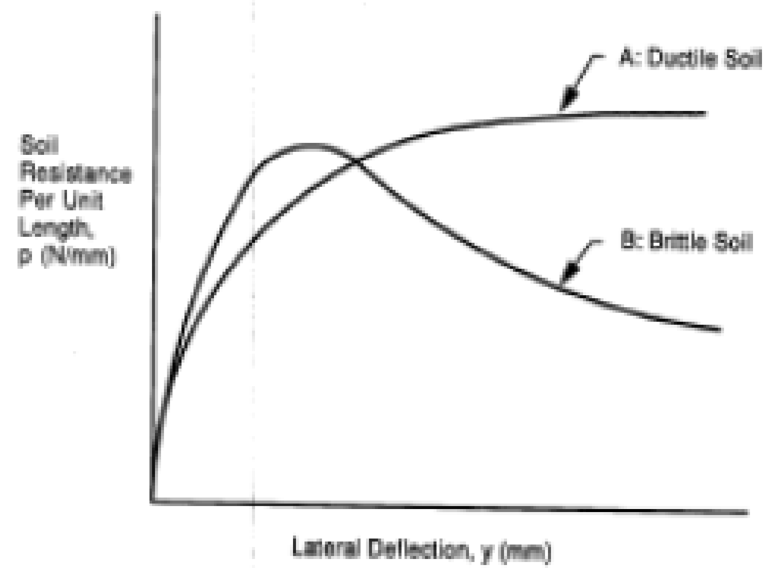
The Genesis of the P-Y Curve:



(Reese and Van Impe, 2001)

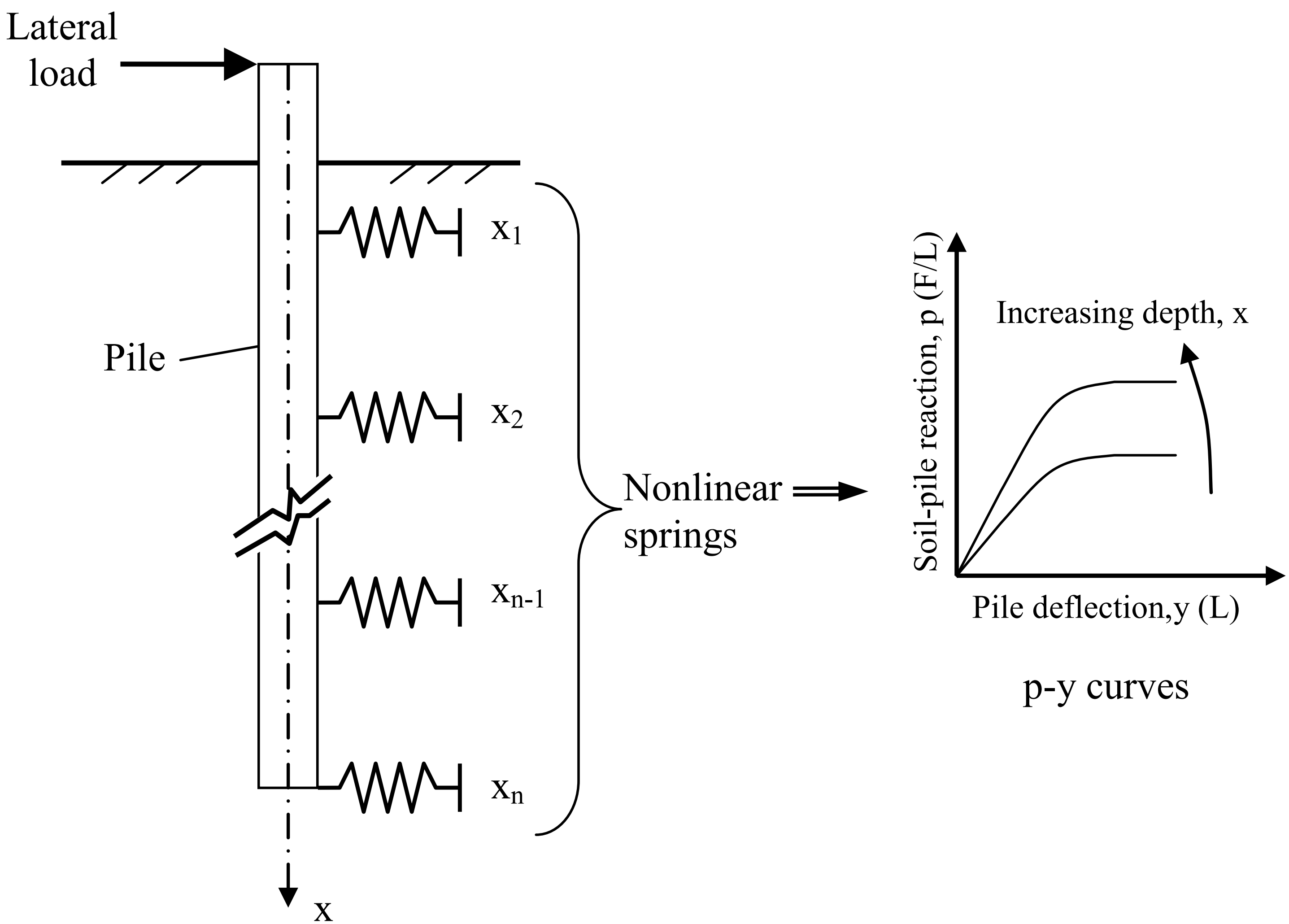
P-v curve Method





P-Y CURVES

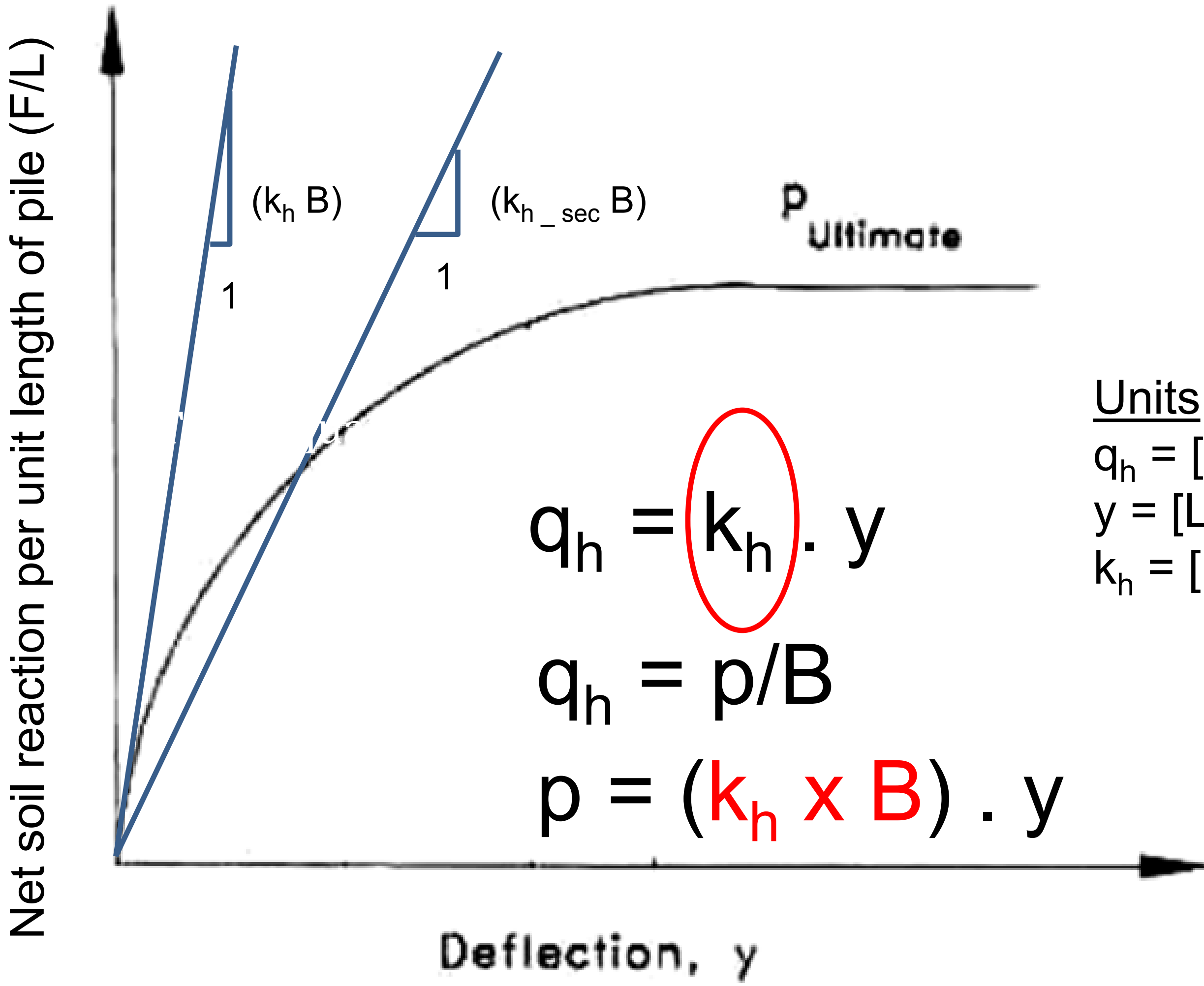
p-y model used for analysis of laterally loaded piles



Careful with confusing terminology:

- Horizontal modulus of subgrade reaction (k_h): relates lateral pressure $q_h = k_h \times y$ [units: F/L^3]
- Subgrade reaction modulus (K): $p = K \times y$ [units: F/L^2] ← $K = k_h \times B$
- Coefficient of subgrade reaction (n_h): rate of increase of subgrade reaction modulus (K) with depth (z): $K = n_h \times z$ [units: F/L^3]

Soil reaction (p-y curve) and Horizontal Modulus of Subgrade Reaction (k_h)



$$q_h = k_h \cdot y$$

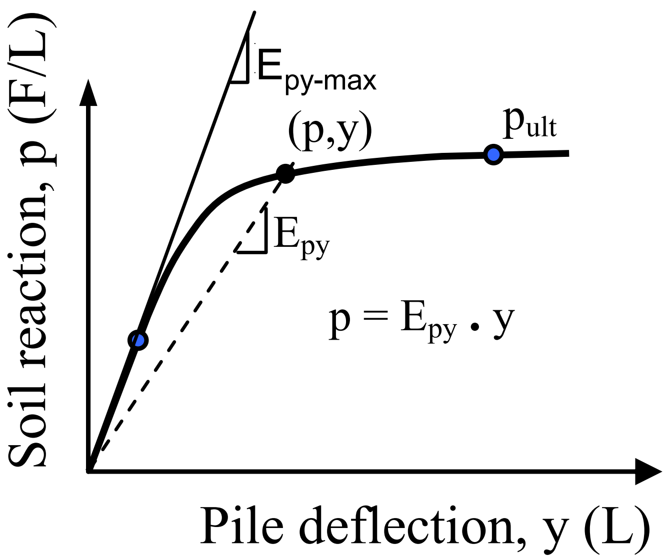
$$q_h = p/B$$

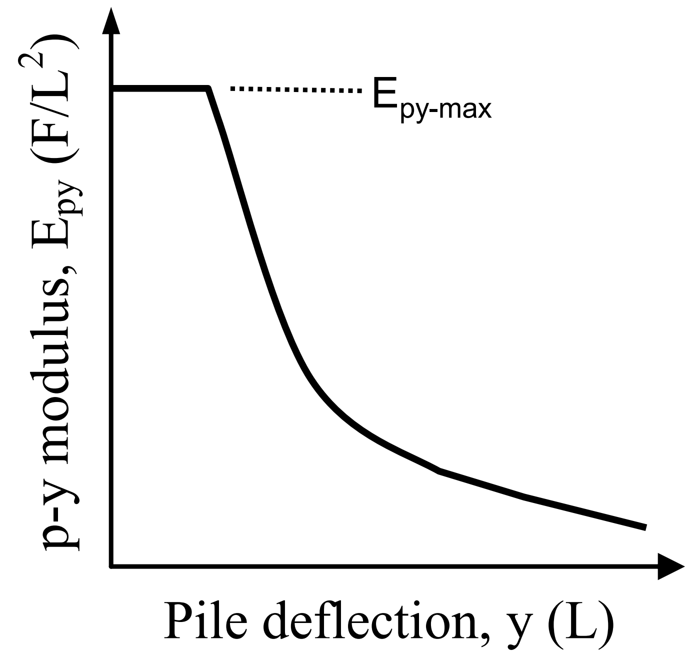
$$p = (k_h \times B) \cdot y$$

Units:
$q_h = [F/L^2]$
$y = [L]$
$k_h = [F/L^3]$

Careful units of k_h same as k_v (F/L^3) but genesis is different

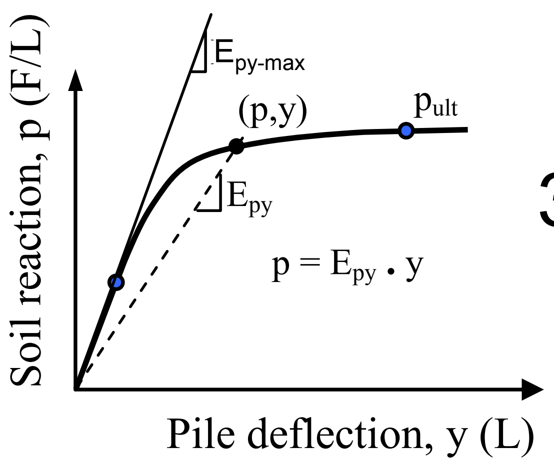
Elements of a p-y curve





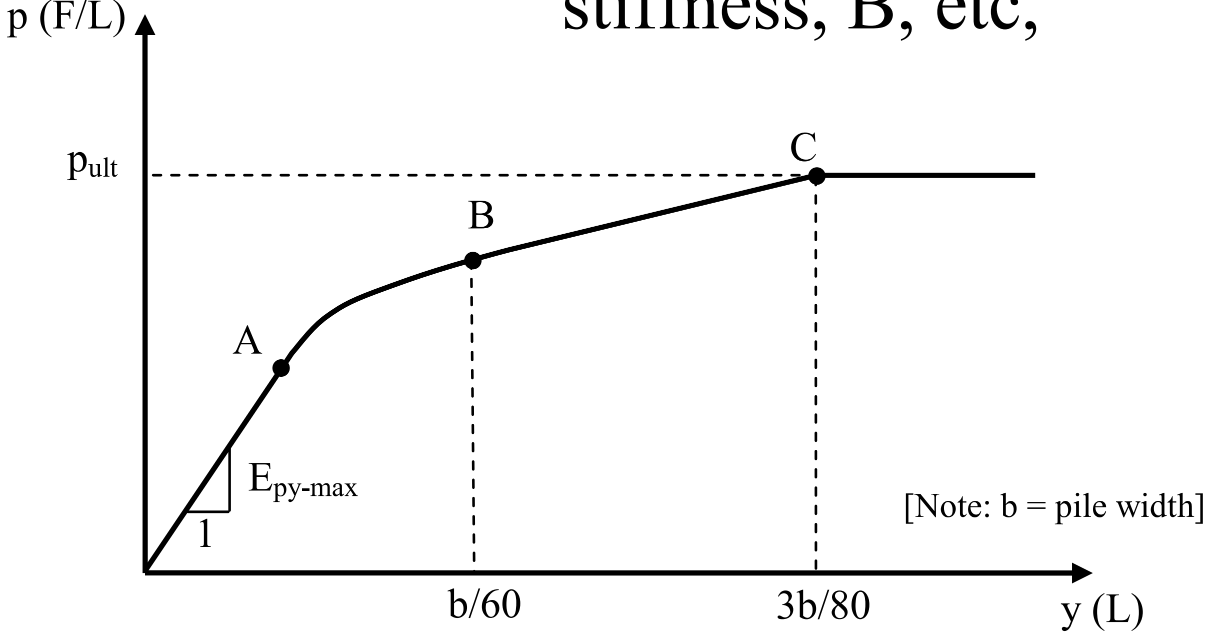
Elements of a p-y curve

1. Initial slope $E_{py\text{-}max}$:
   - Considerable scatter of reported values.
   - Most $E_{py\text{-}max} = k_h \times B$
   - Some P-Y curves have $E_{py\text{-}max} \rightarrow \infty$
2. P_{ult} (asymptotic value):
   - From ultimate load theories (e.g., Broms, 1964):
     - Clays: $9S_uB$;
     - Sands: $3K_p\sigma'_vB$ or $K_p^2\sigma'_vB$
3. Transition curve(s) from origin to P_{ult}.

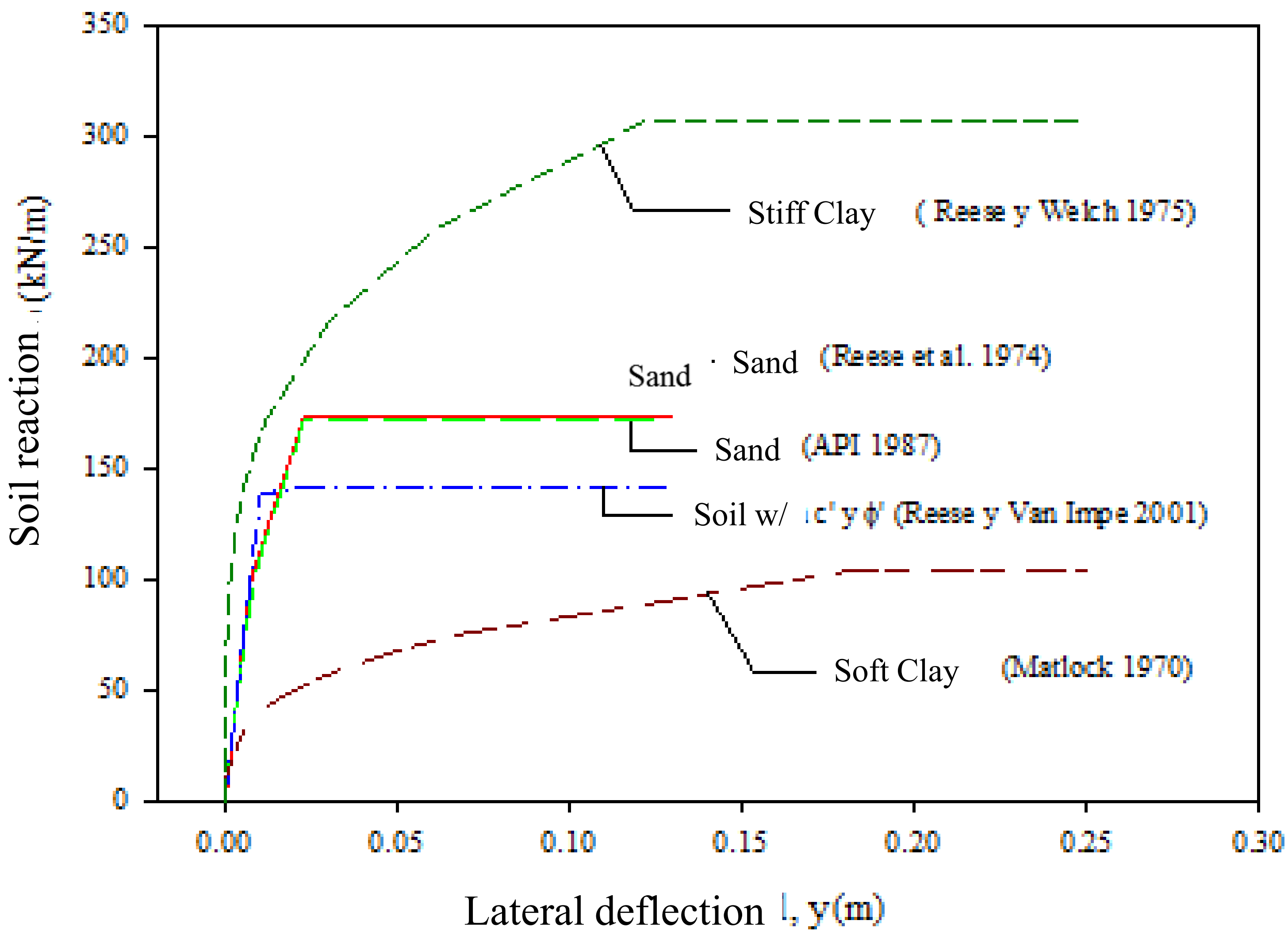


Example p-y curves in Sands
(Reese et al. 1974)

- $P_{ult} = f(\phi$, stress level)
- $E_{py\text{-}max}$: related to soil stiffness, B, etc,



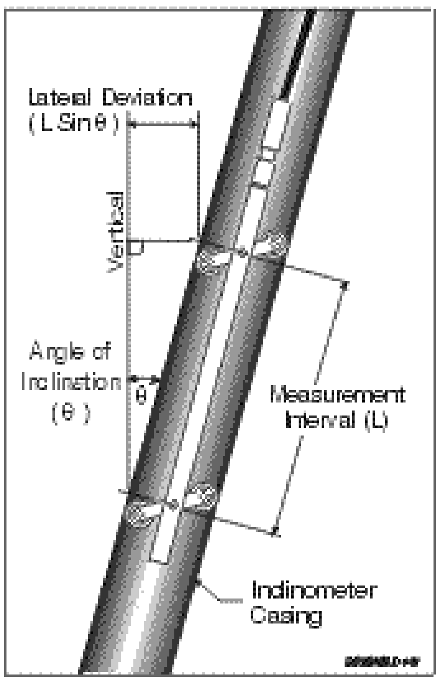
P-Y Curves for Different Soil Types

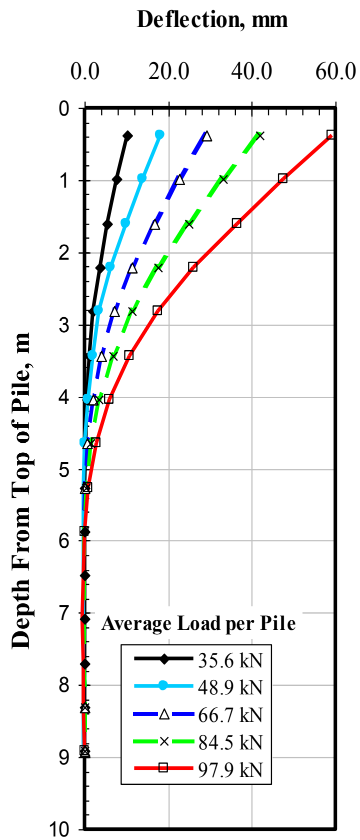


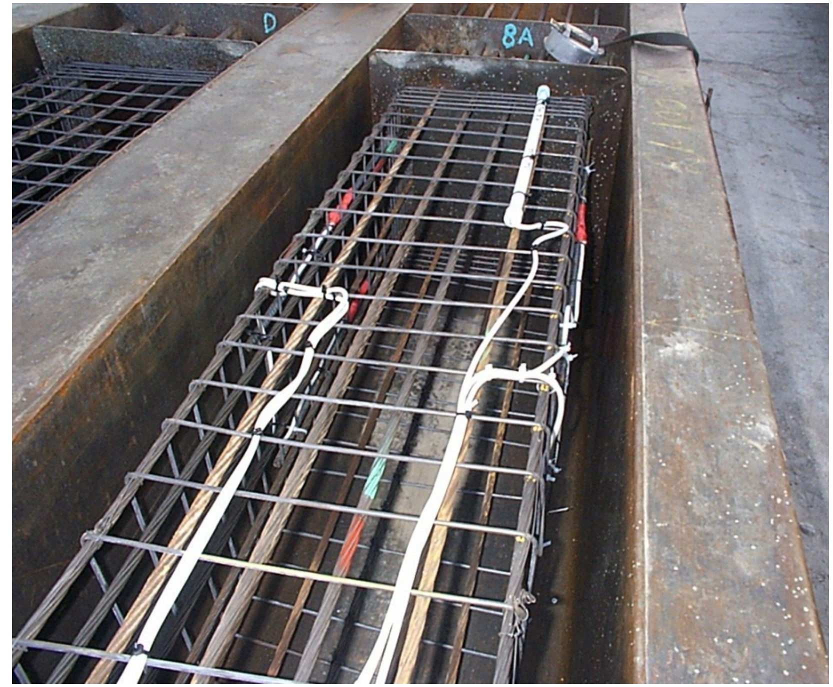
Experimental P-Y Curves

- Lateral load tests on instrumented piles
- Vey few high quality tests are available
- Basis for P-Y Curves proposed in the literature
- Typically from deflected shape measurements (e.g., inclinometers)
- Better if from Moment (or curvature) measurements using closely spaced pairs of strain gages (very few of these)

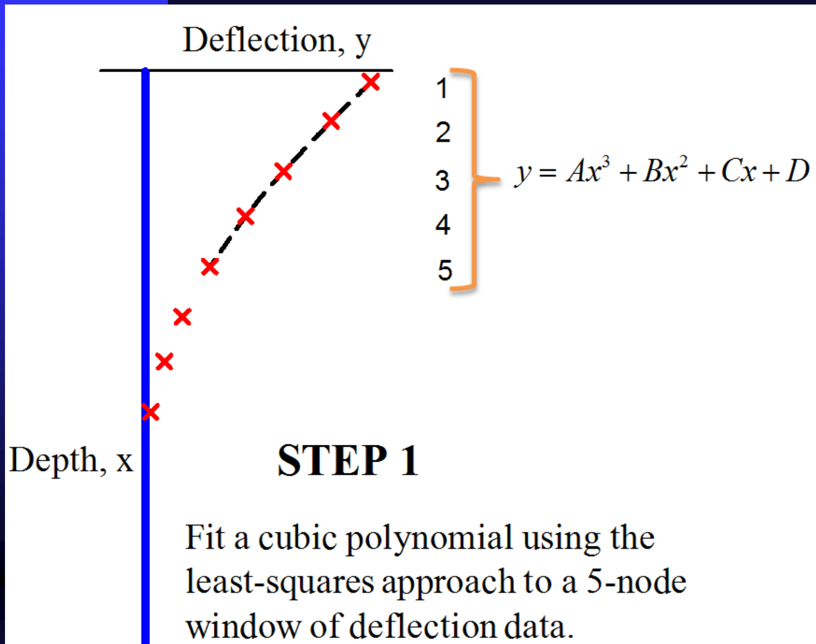
Additional Instrumentation

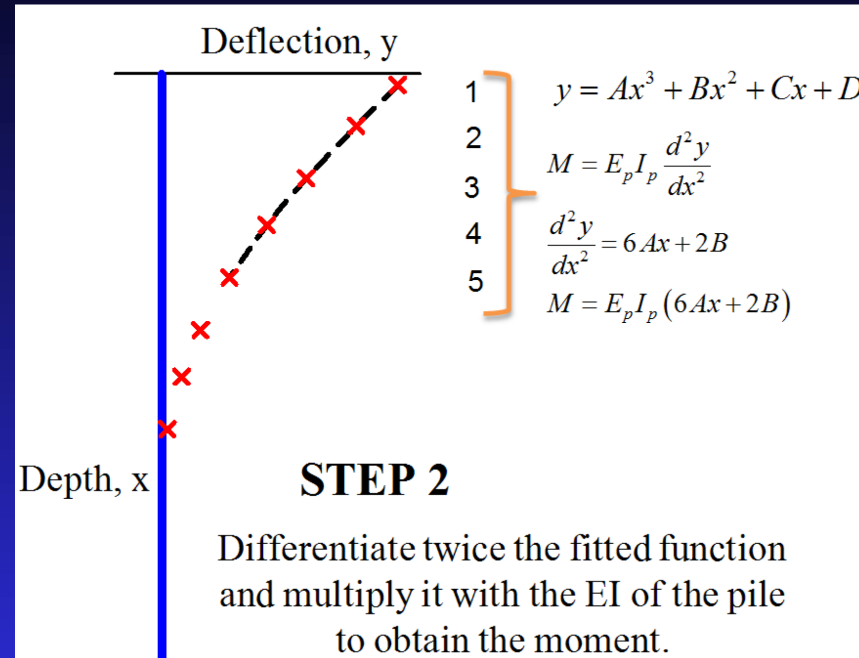


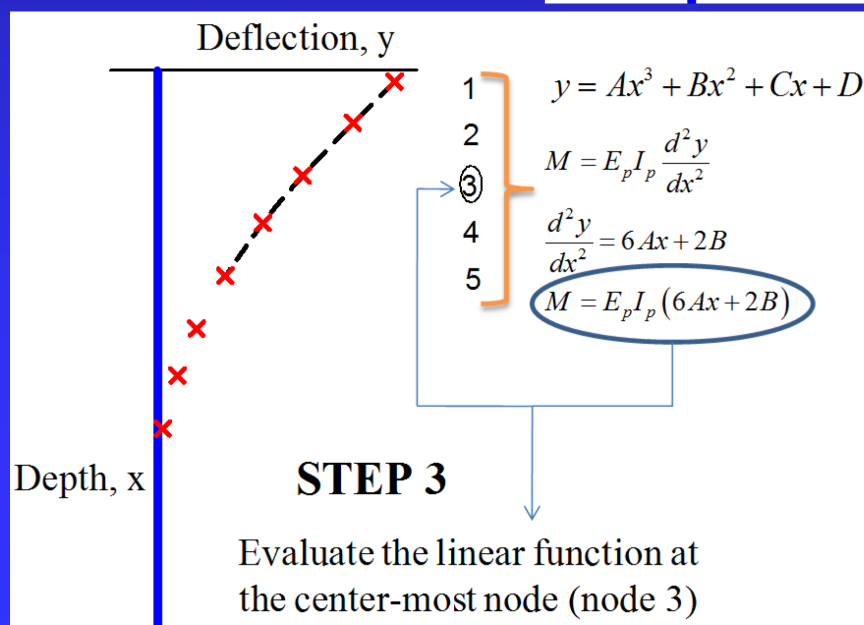




P-Y Curves from Experiments







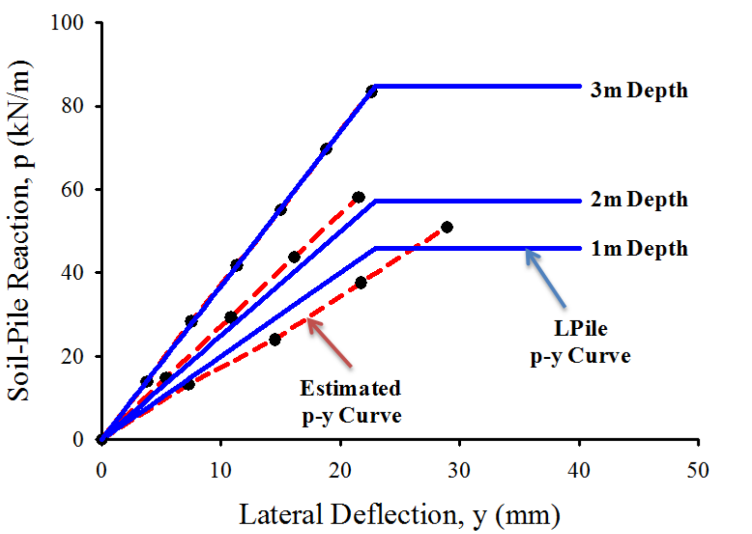
P-Y Curves from Experiments

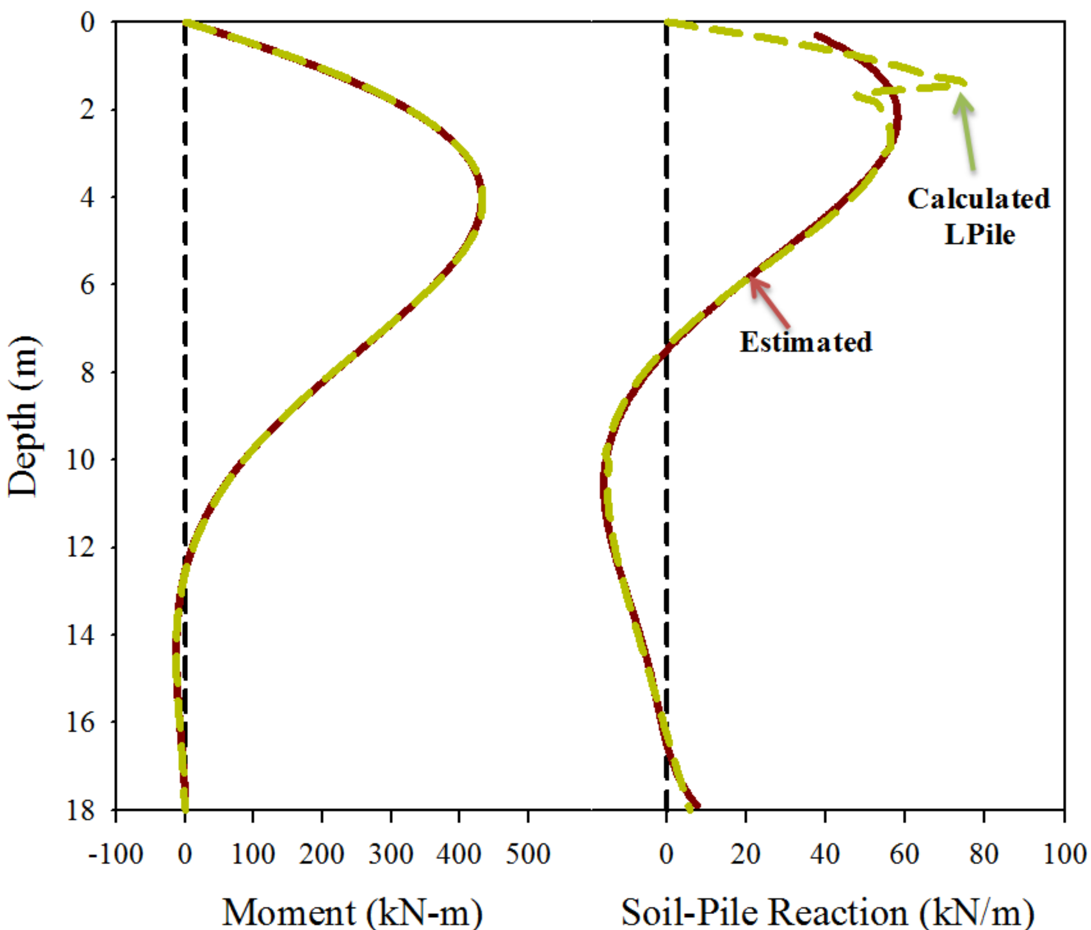
STEP 6

Make an average of the moments obtained on steps 3, 4 and 5, and assign it to the center-most node in step 4

Recall from beam theory:

$$p = \frac{d^2M}{dx^2}$$





Commonly used p-y curves for different soils

Soil Type and Condition	Reference
Soft clay below the water table	Matlock (1970)
Stiff clay below the water table	Reese, Cox, and Koop (1975)
Stiff clay above the water table	Welch and Reese (1972), Reese and Welch (1972)
Sands	Reese, Cox, and Koop (1974)
Sands	API RP2A (1991)
Soils with cohesion and friction	Evans and Duncan (1982)
Weak rock	Reese (1997)
Strong rock	Nyman (1982)

(adapted from Reese and Isenhower, 1997)

ANALYTICAL METHODOLOGY

Software for p-y based analysis:

- Solve beam equation with finite difference or finite elements
- COM624
- LPILE
- FB-Pier (FB-Multipier)
- Matlab or Mathcad spreadsheets

Other Methodologies

- Strain Wedge Model
- FEM
- Characteristic Load Method (LPILE based)

POSSIBLE LIMITATIONS OR CHALLENGES

Potential Limitations P-Y Curves

- The soil is idealized as a series of independent nonlinear springs represented by p-y curves. Therefore, the continuous nature of the soil is not explicitly modeled.

Potential Limitations P-Y Curves

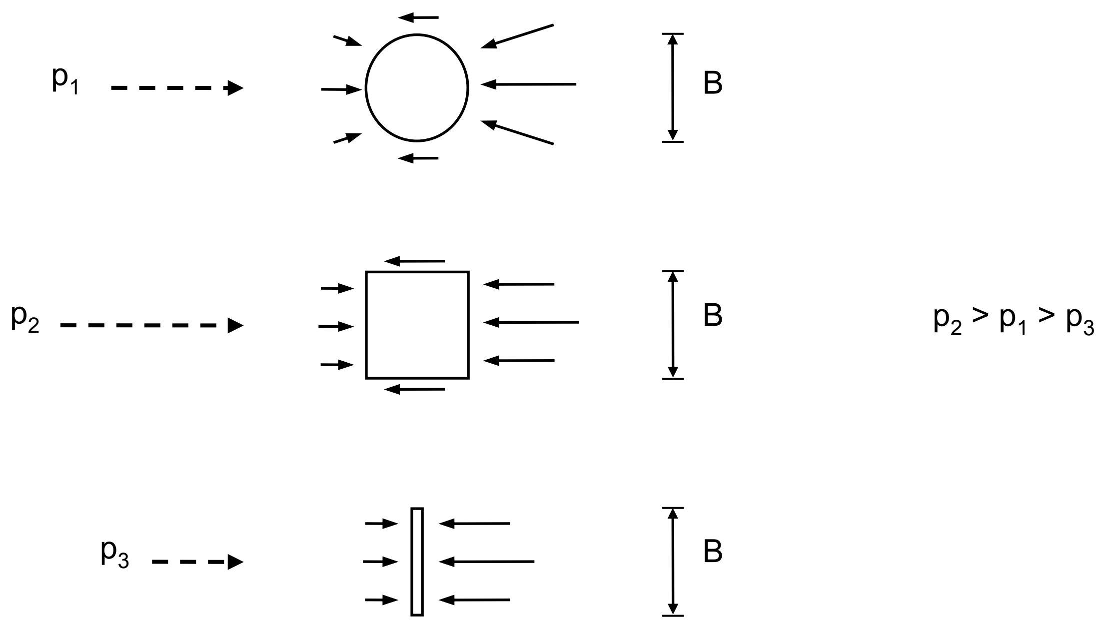
- The results are very sensitive to the p-y curves used. ***The selection of adequate p-y curves is the most crucial problem when using this methodology to analyze laterally loaded piles*** (Reese and Van Impe 2001).
- P-Y curves in literature are empirical in nature. Need to carefully review applicability of the selected curves.

On selection of appropriate p-y modulus and p-y curves

- Important and difficult task.
- Selection of values of initial p-y modulus, $E_{py\text{-}max}$, although related to the soil modulus, is also related to the interaction between the pile and the soil.
- Reese and Van Impe (2001) point out that p-y curves and modulus are influenced by several pile related factors, such as:
    - Pile type and flexural stiffness,
    - Type of loading (monotonic or cyclic),
    - Pile geometry,
    - Pile cap conditions, and
    - Pile installation conditions.

Potential Limitations (Continued) Cross section of pile

- Most P-Y curves only depend on pile width (B). Shape or Depth is not explicitly included in P-Y curves currently in the literature.



a) Pilote de Acero y Sección Sólida

I_p = 0.0004 m^2

E_p = 200 GPa

E_pI_p = 8.5E4 kN-m^2 (constante)

b) Pilote Tubular de Acero con t = 0.05 m

I_p = 0.00034 m^2

E_p = 200 GPa

E_pI_p = 6.8E4 kN-m^2 (constante)

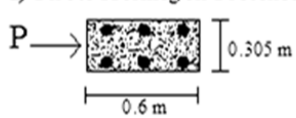
c) Pilote H de Acero

I_p = 0.004 m^2

E_p = 200 GPa

E_pI_p = 8.2E5 kN-m^2 (constante)

d) Pilote Hormigón Pretensado



I_p = 0.0055 m^2

E_p = 26.5 GPa

E_pI_p = 1.5E5 kN-m^2 (no lineal)

e) Pilote de Plástico Reforzado

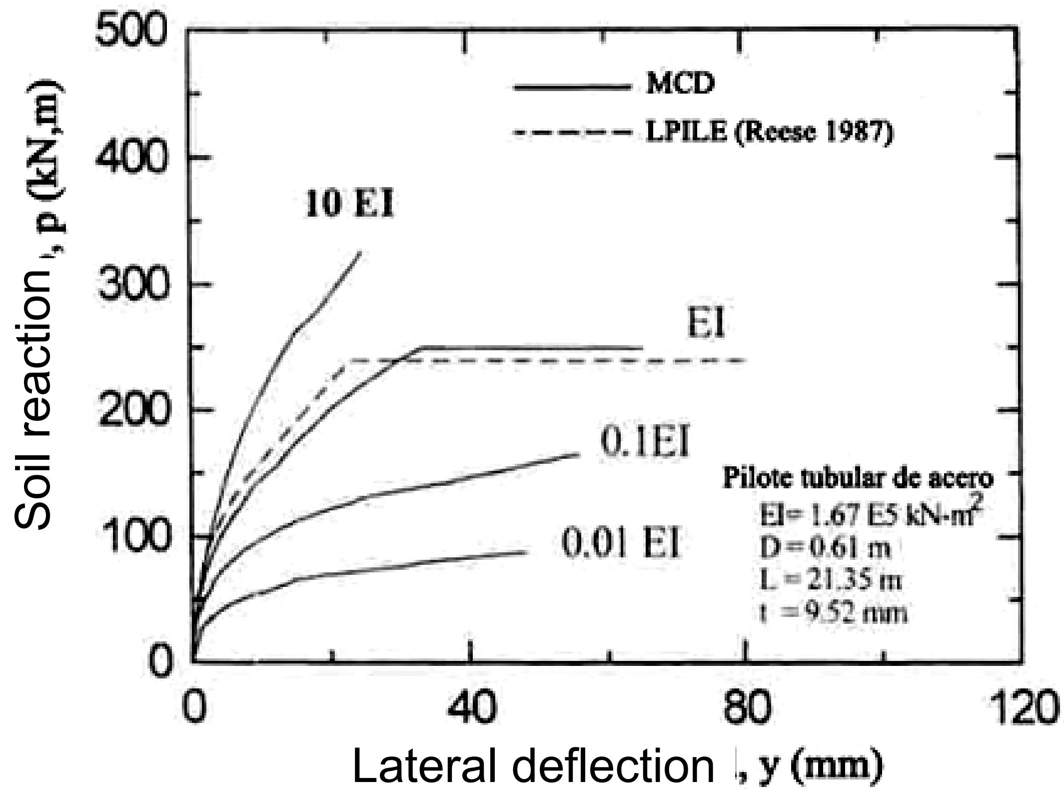
I_p = 0.00034 m^2

E_p = 275 MPa

E_pI_p = 1.2E2 kN-m^2 (no Lineal)

Potential Limitations (Continued) Flexural Stiffness (EI) of pile

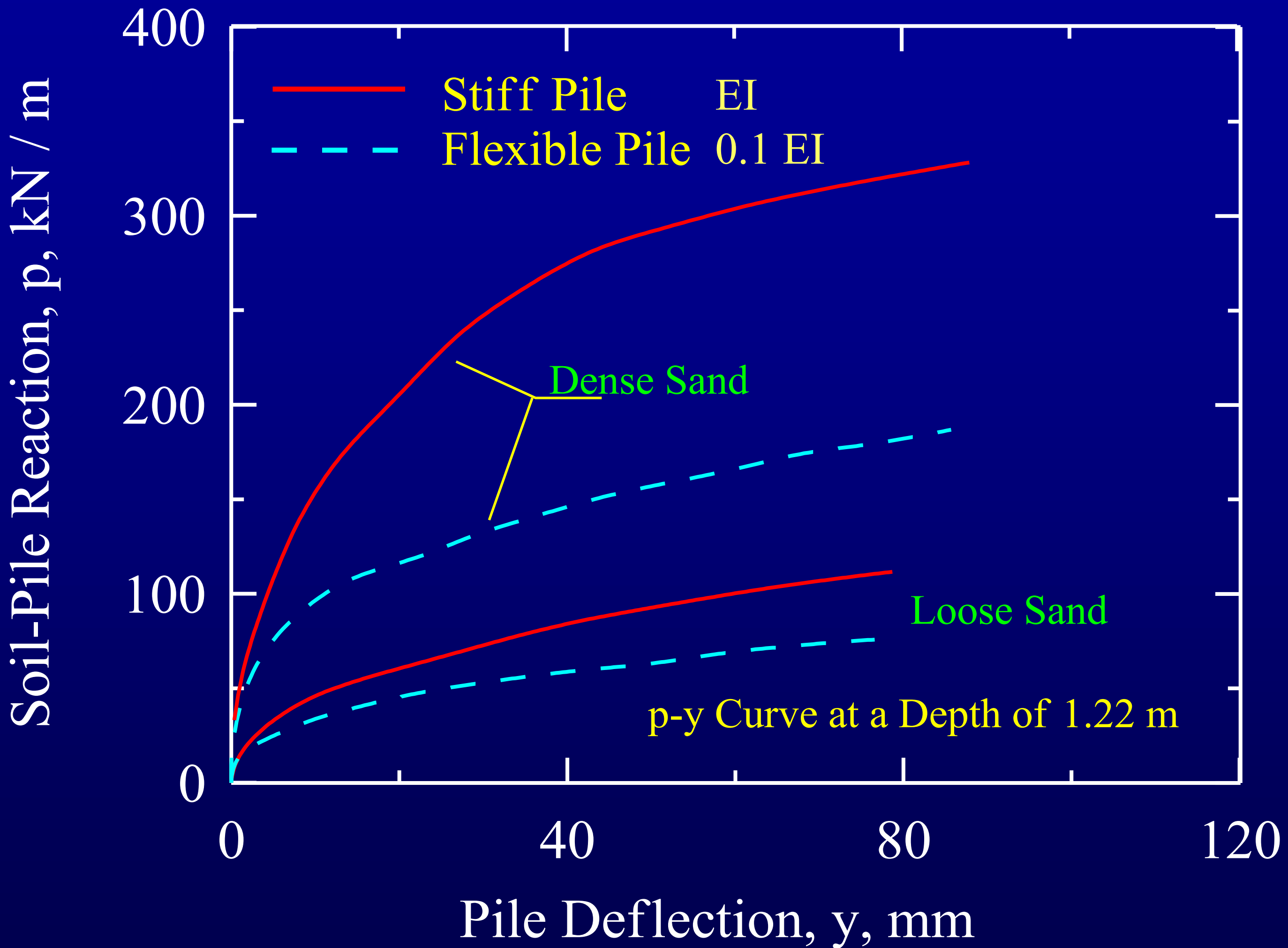
- P-Y curves don’t directly incorporated effects of EI of pile (Only in pile model).



Ashour and Norris (2000)

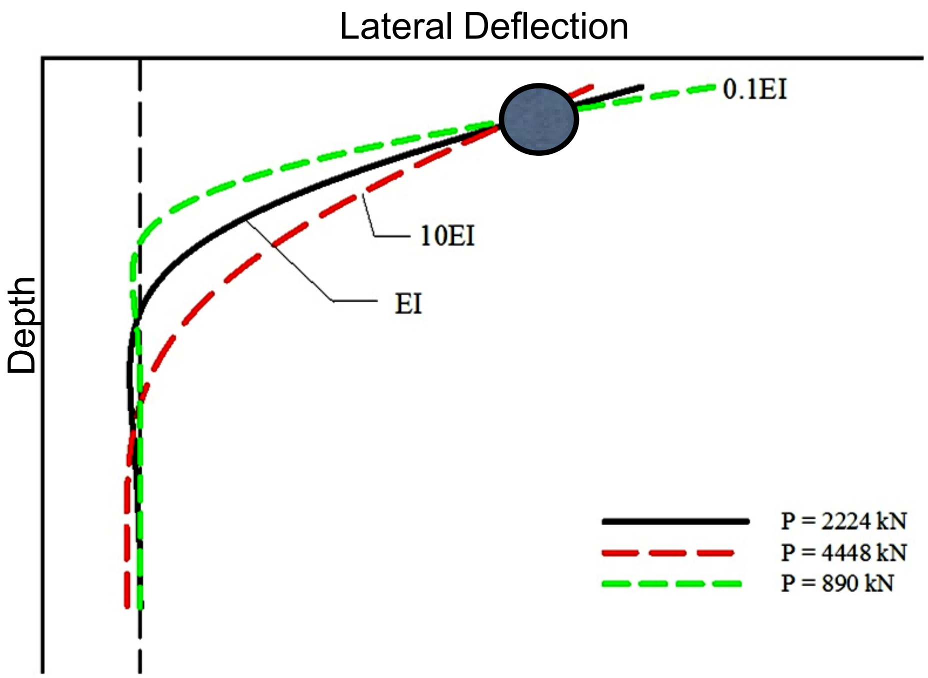
The traditional p-y curve (in LPILE) does not account for the pile/shaft EI variation

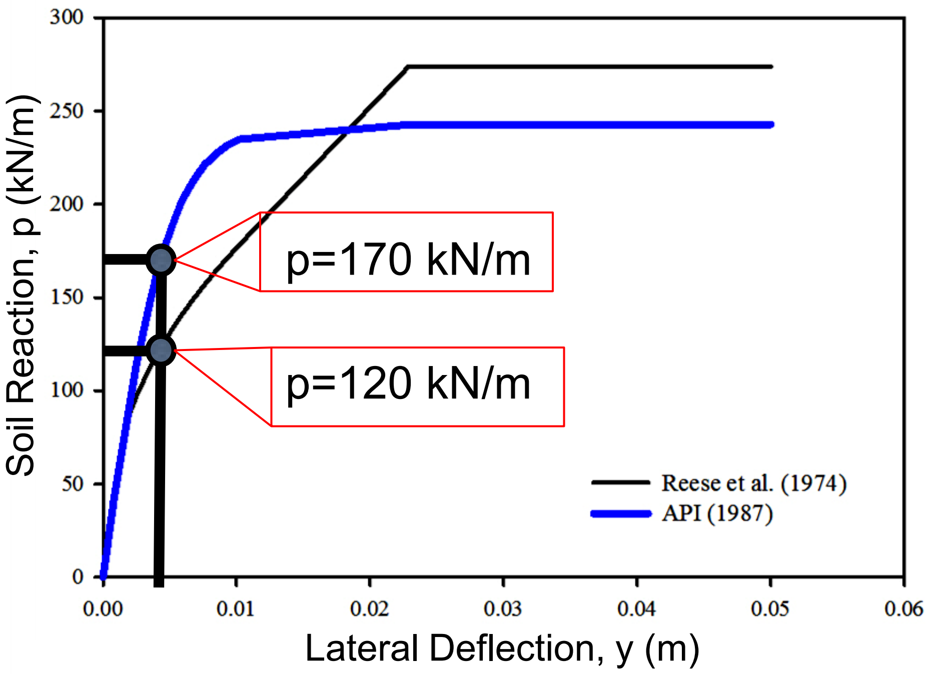
Based on the Strain Wedge Model Analysis



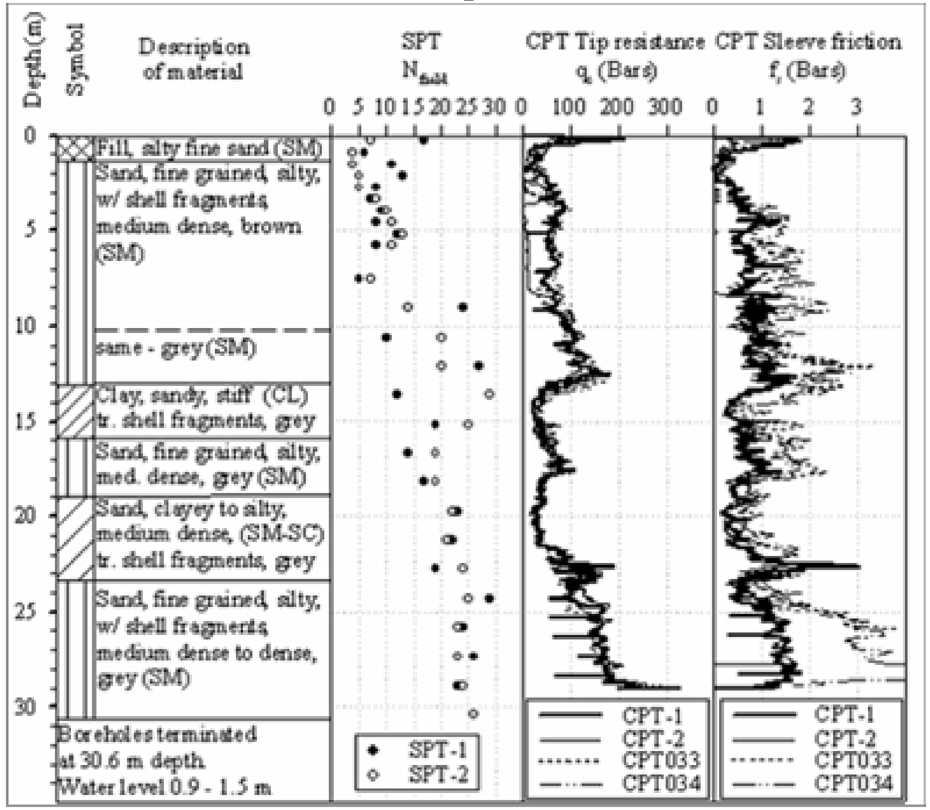
Effect of Pile Bending Stiffness on the p-y Curve in Sand

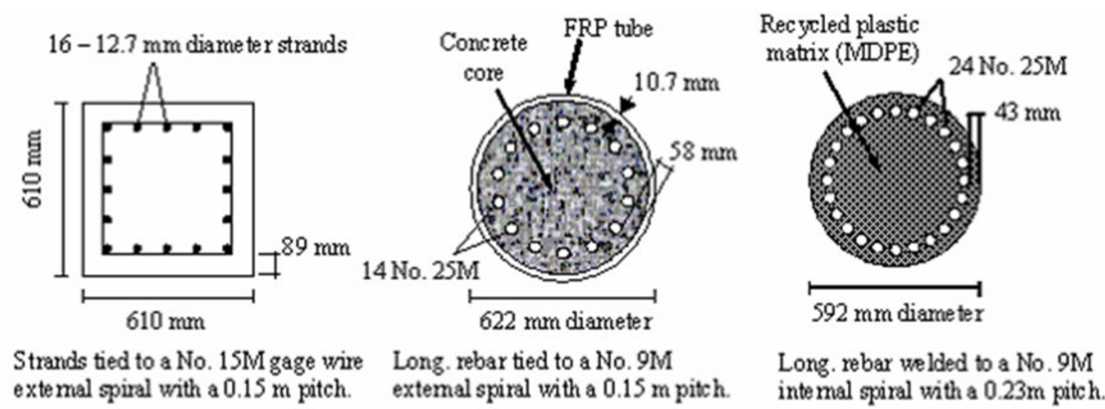
EI Effects


Lateral Deflection
Depth
0.1EI
10EI
EI
P = 2224 kN
P = 4448 kN
P = 890 kN


Soil Reaction, p (kN/m)
Lateral Deflection, y (m)
300
250
200
150
100
50
0
0.00
0.01
0.02
0.03
0.04
0.05
0.06
p=170 kN/m
p=120 kN/m
Reese et al. (1974)
API (1987)

Route 351 Bridge Case History





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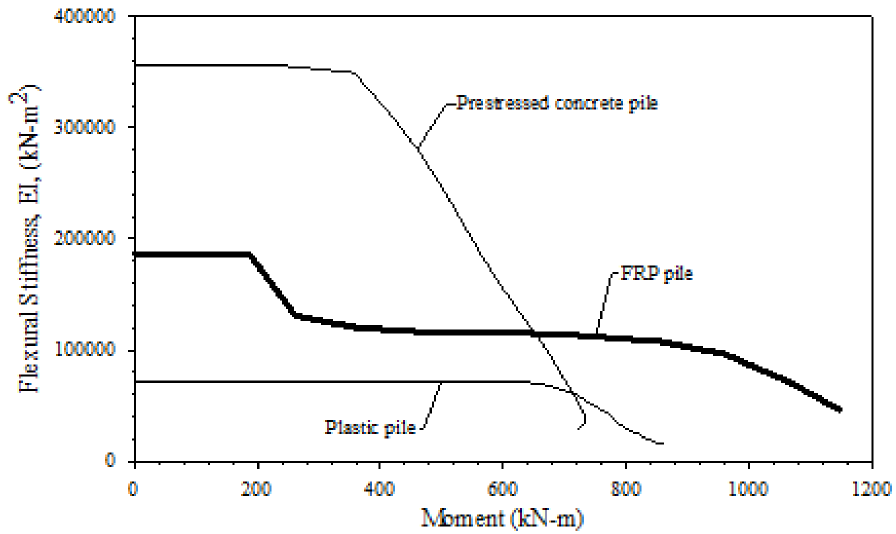
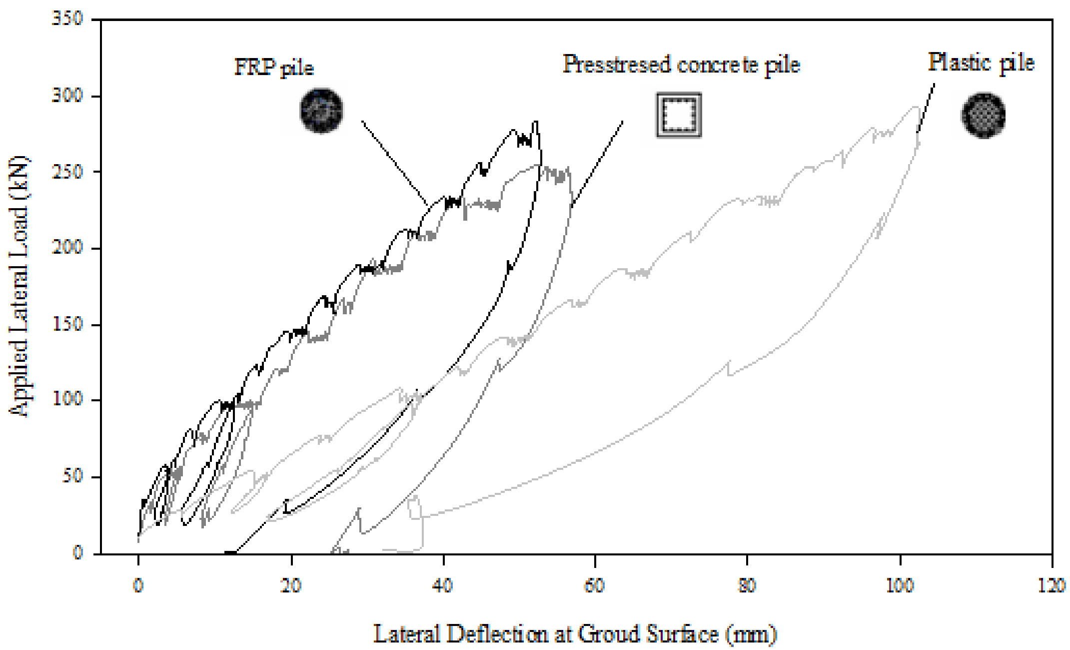
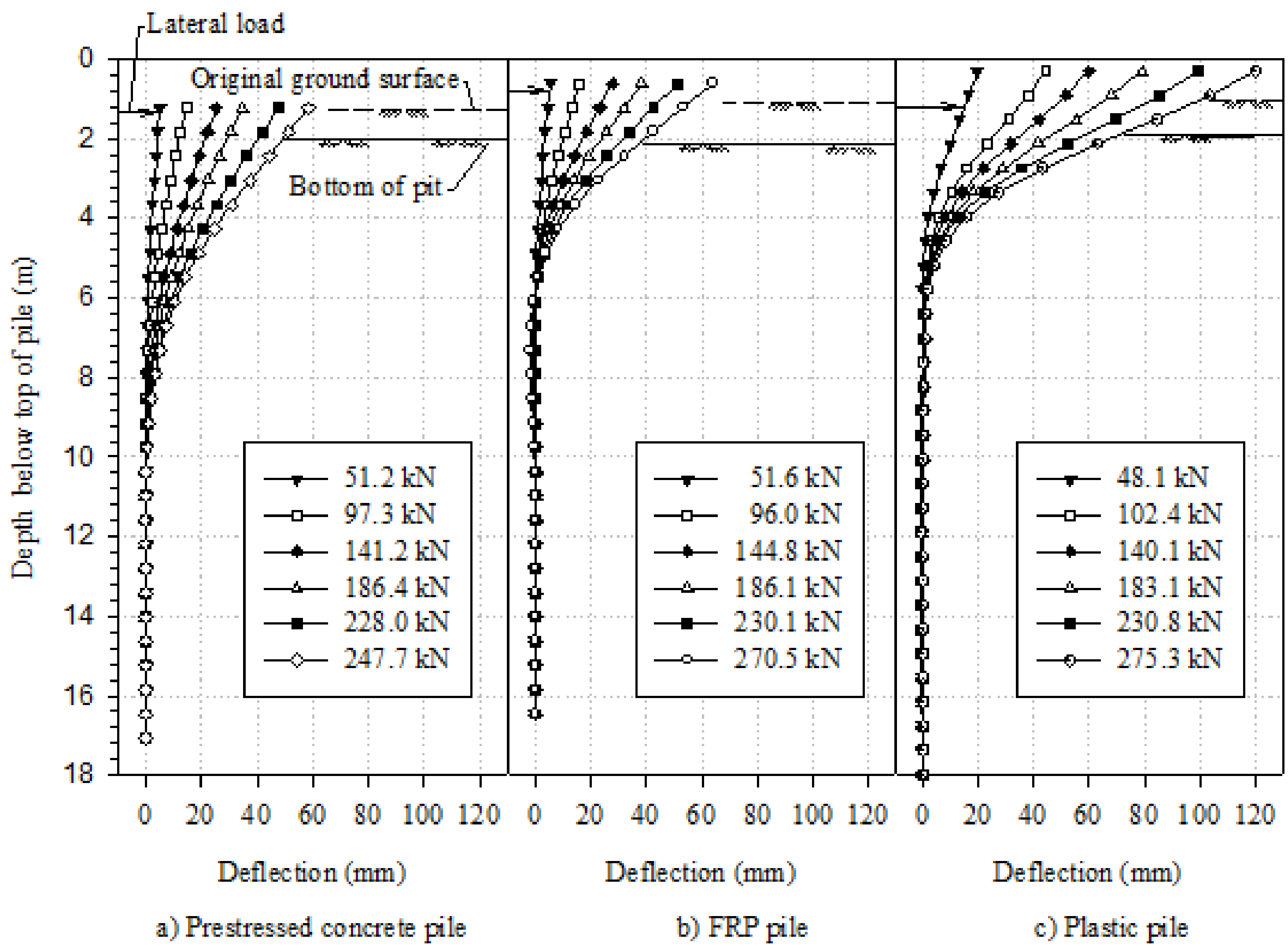




Route 351 Bridge Case History



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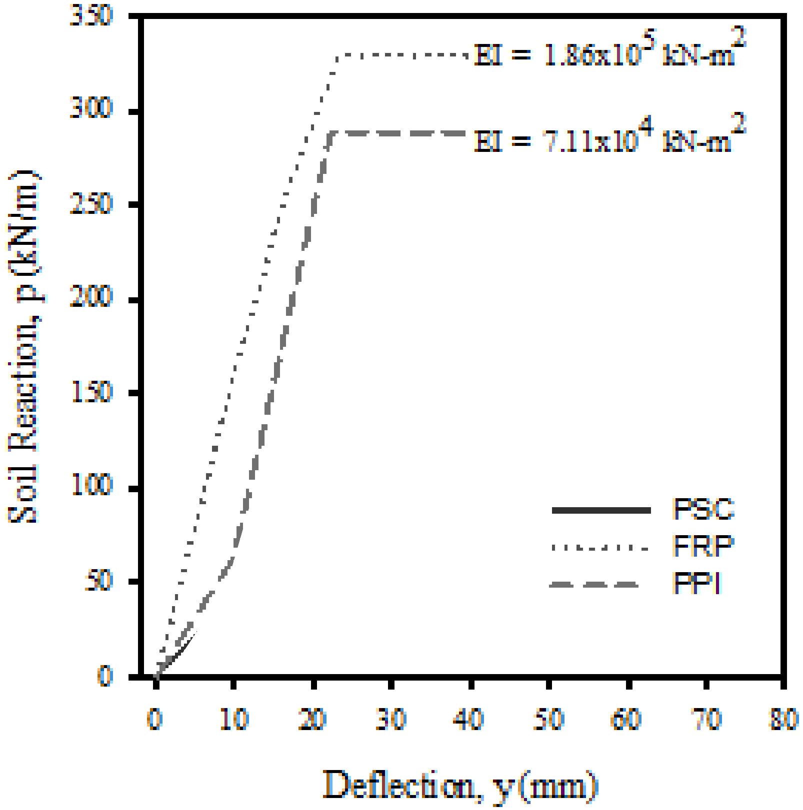
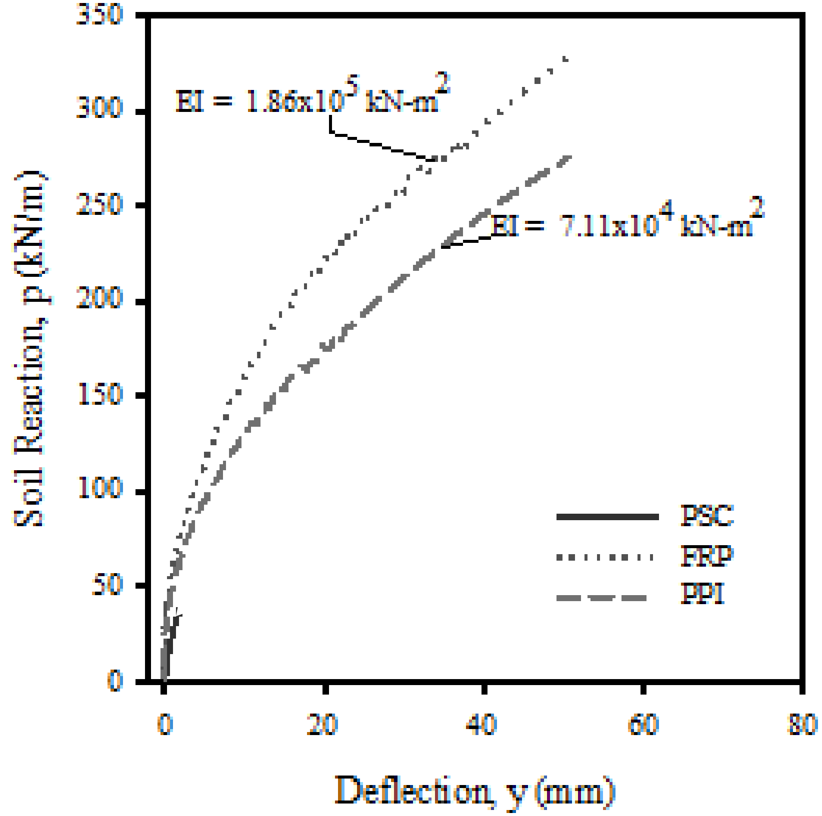
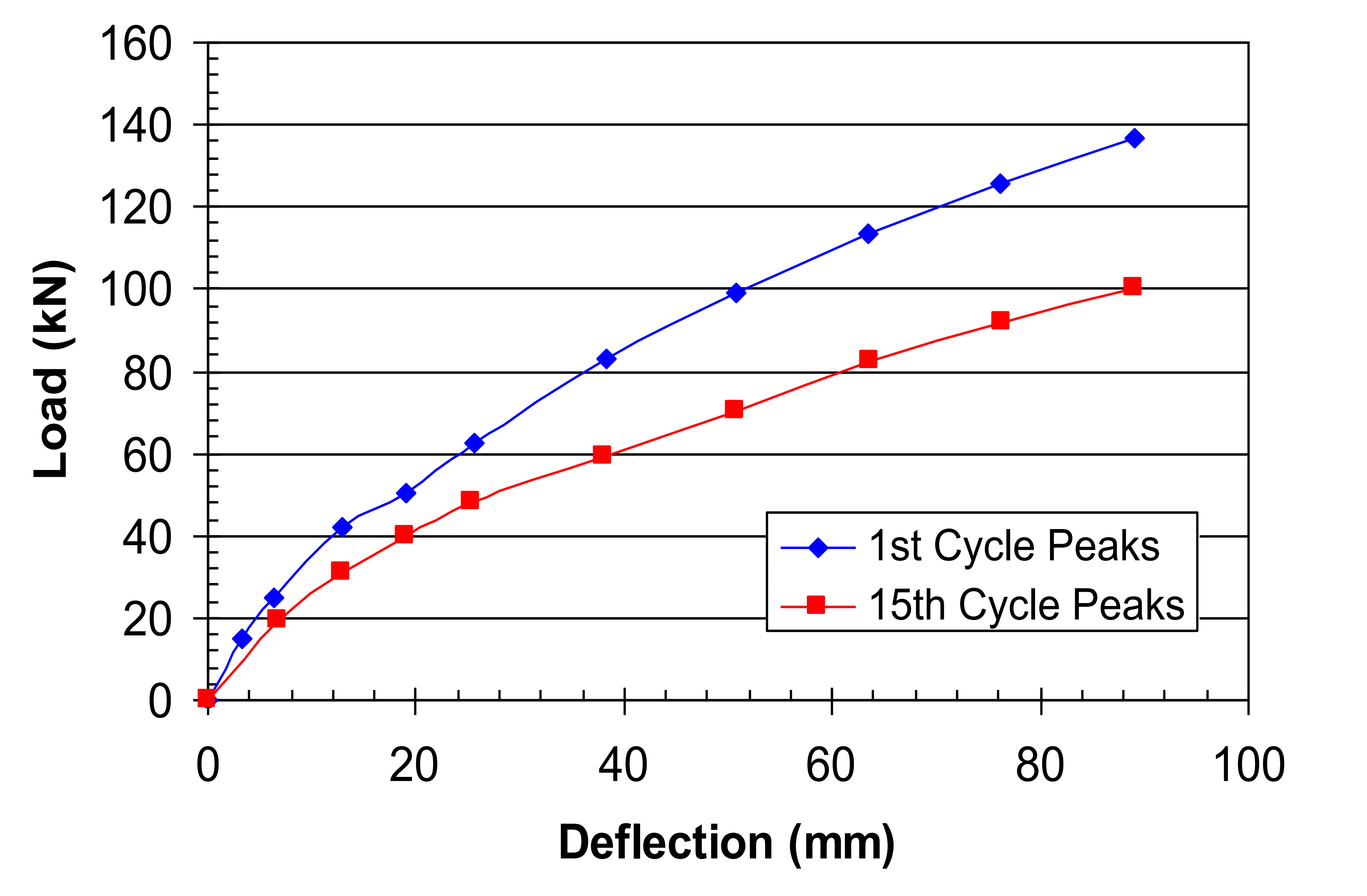




Lateral Cyclic loading on Piles

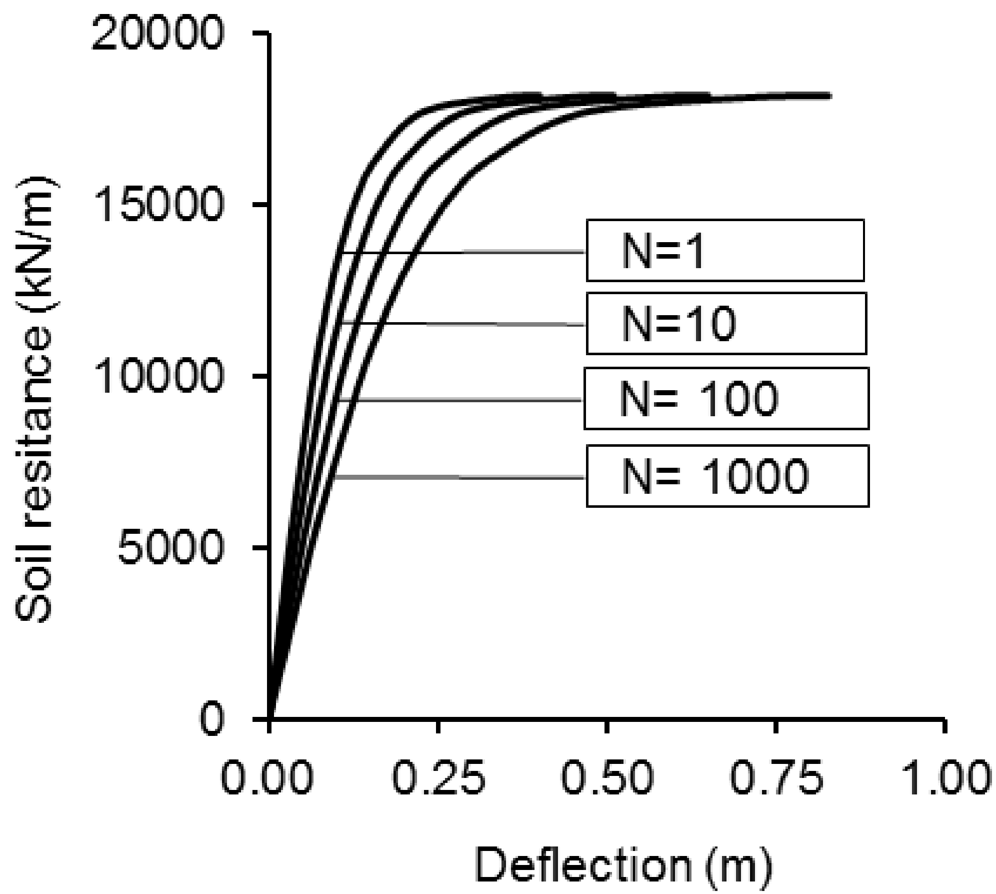
- Limited experimental data.
- API P-Y curves for sands suggest incorporating 10% degradation of p-y curve for offshore piles.
- A few experimental studies developed cyclic P-Y curves

Effect of Cyclic Loading



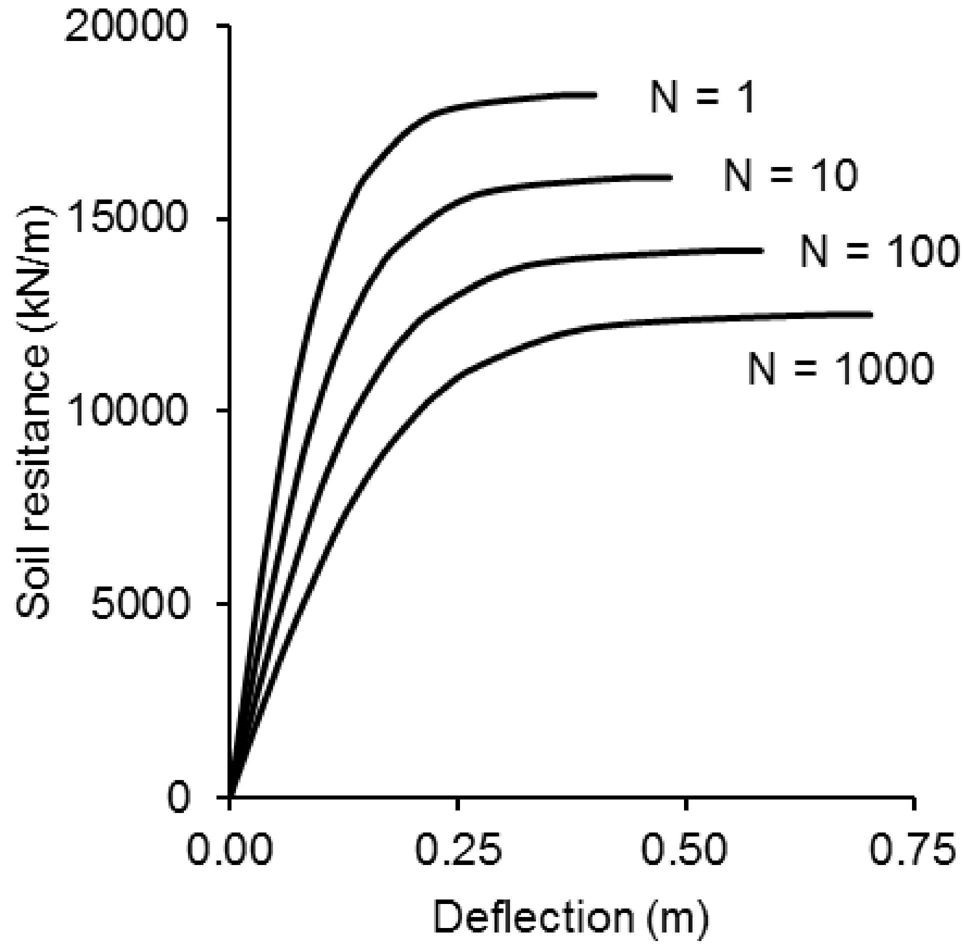
Cyclic P-Y Curves by Little and Briaud (1988):

- Most experiments up to 20 lateral load cycles.
- $y_N = y_1 . N^a$



Cyclic P-Y Curves by Long and Vanneste (1994):

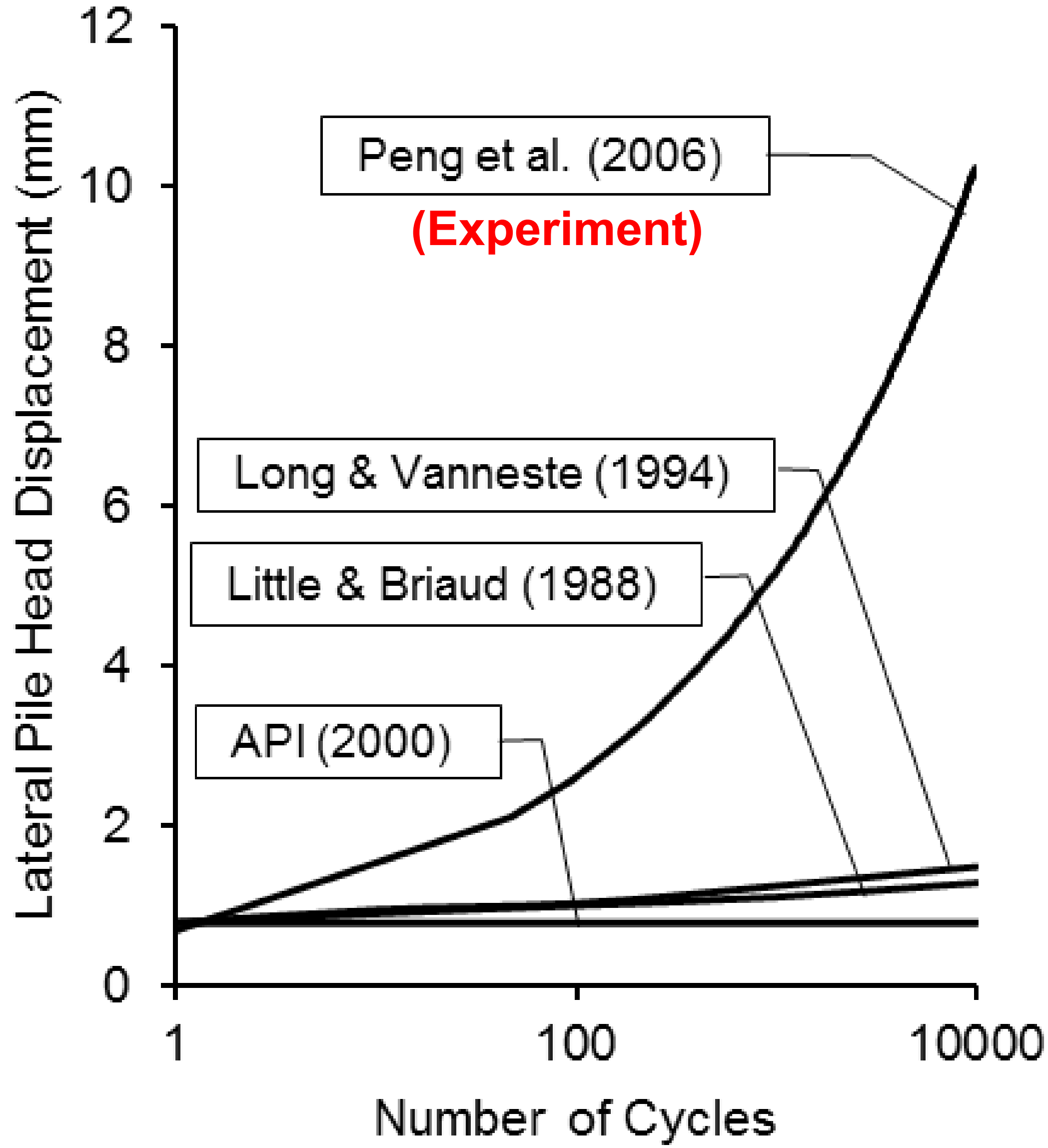
- 34 experiments (some up to 500 lateral load cycles).
- Modified P_n and Y_n
- $P_N = P_1 . N^{-0.4t}$
- $y_N = y_1 . N^{0.6t}$



Possible limitation with cyclic loading

- Little experience and scarce availability of experimental data.
- Available experiments very few load cycles.
- Wind action on highway signs, sound barrier foundations; Or loading on bridge piles (thermal, current, wave, etc) can involve $N > 10^4$ load cycles during pile design life.

Experiments on Model Pile by Peng et al (2006) (N=10^4 cycles)



Summary & Conclusions

- The P-Y Curve based methodology for analysis of laterally loaded piles is easy and reliable
- Empirical in nature, but backed by decades of experience.
- However, several items may still need additional research to overcome some identified possible limitations. (i.e., still room for improvement).
- Also practitioners should be aware of alternative emerging methodologies such as the SWM (Need to incorporate into design tool box) (Several DoT's already using).

THANK YOU!

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