Analyses of Lateral Loaded Piles with P-Y Curves - Observations on the Effect of Pile Flexural Stiffness and Cyclic Loading

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Outline

• Background
  • Beam on Elastic Foundation (BEF)
  • Subgrade Reaction
  • Beam Theory (related to net soil reaction per unit length \( P \) and pile lateral deflection \( Y \))

• P-Y curves
  • Elements of a P-Y curve (preferred terminology)
  • Commonly used P-Y Curves (empirical basis)

• The analytical methodology

• Possible limitations/challenges
  • Influence of pile cross section and EI?
  • Effects of lateral load cycles?

• Summary and Conclusions
BACKGROUND
Beam on Elastic Foundation (BEF)

• After Winkler (1867) aka as Beam on Winkler Foundation (BWF)

$k_h = \text{Modulus of Lateral (Horizontal) Subgrade Reaction} \ [\text{F/L}^3]$

Typically linear representation of soil reaction.
Vertical Modulus of Subgrade Reaction

- Modulus of subgrade reaction (k)
- Is it a soil property? **NO**
- Footing, rafts, pavement design
- Westergaard's work in 1920's
- \( q = k \Delta \) (used by structural engineers)
  - \( q \) = applied or contact pressure [F/L^2]
  - \( \Delta \) = settlement of footing under q [L]
- \( k = q/\Delta = \) slope (linear spring constant) = modulus of subgrade reaction [F/L^3]
Vertical Modulus of Subgrade Reaction (Continued)

- Elastic settlement of circular footing:
  \[ \Delta = I \frac{q.B.(1-\mu^2)}{E_s} \]
- \( \mu = 0.5 \) (undrained) \( \sim 0.3 \) (drained)
- \( E_s = E_u \) (undrained) vs \( E' \) (drained)
- \( k_v = q/\Delta = \frac{E_s}{I.B.(1-\mu^2)} \)

- **Not a fundamental soil property**
- Not readily measured
- Depends on many factors such as size and shape of footing, type of soil, relative stiffness of footing and soil, vary along footing, vary with time, etc.
Beam theory

\[ S = \frac{dy}{dx} \]
\[ M = E_pI_p \frac{d^2y}{dx^2} \]
\[ V = E_pI_p \frac{d^3y}{dx^3} \]
\[ p = E_pI_p \frac{d^4y}{dx^4} \]

\[ n = \frac{d^2M}{dx^2} \]

**Where:**
- \( E_pI_p \) = Flexural stiffness of pile;
- \( y \) = Lateral deflection;
- \( x \) = Depth below the pile head;
- \( p \) = Soil reaction per unit length of pile.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance along the length of the pile (measured from pile head)</td>
<td>$x$</td>
<td>[L]</td>
</tr>
<tr>
<td>Distance to neutral axis within pile cross section</td>
<td>$z$</td>
<td>[L]</td>
</tr>
<tr>
<td>Deflection</td>
<td>$y$</td>
<td>[L]</td>
</tr>
<tr>
<td>Slope or rotation of pile section</td>
<td>$\phi = \frac{dy}{dx}$</td>
<td>[Dimensionless]</td>
</tr>
<tr>
<td>Curvature</td>
<td>$\kappa = \frac{d^2y}{dx^2}$</td>
<td>[Radians/L]</td>
</tr>
<tr>
<td>Bending moment</td>
<td>$M = E_p I_p \cdot \frac{d^2y}{dx^2} = E_p I_p \cdot \kappa$</td>
<td>[F x L]</td>
</tr>
<tr>
<td>Shear force</td>
<td>$V = E_p I_p \cdot \frac{d^3y}{dx^3}$</td>
<td>[F]</td>
</tr>
<tr>
<td>Axial load</td>
<td>$Q$</td>
<td>[F]</td>
</tr>
<tr>
<td>Soil reaction (or load intensity)</td>
<td>$p = E_p I_p \cdot \frac{d^4y}{dx^4}$</td>
<td>[F/L]</td>
</tr>
</tbody>
</table>

Notes: $E_p I_p =$ flexural stiffness of pile, where $E_p =$ elastic modulus of pile material, and $I_p =$ moment of inertia of pile cross section with respect to the neutral axis.
Relationships between variables

Figure 8.1 Laterally loaded pile problem

- a) Pile loading
- b) Net soil reaction
- c) Pile deflection
- d) Slope
- e) Bending moment

\[ p \]
\[ M + dM \]
\[ V + dV \]
The Genesis of the P-Y Curve:

(Reese and Van Impe, 2001)
P-v curve Method
P-Y CURVES
p-y model used for analysis of laterally loaded piles

![Diagram showing p-y model](image)

- Lateral load
- Pile
- Nonlinear springs
- Soil-pile reaction, p (F/L)
- Pile deflection, y (L)
- Increasing depth, x

Increasing depth, x

Soil-pile reaction, p (F/L)

Pile deflection, y (L)

p-y curves
Careful with confusing terminology:

- Horizontal modulus of subgrade reaction ($k_h$): relates lateral pressure $q_h = k_h \times y$ [units: F/L$^3$]
- Subgrade reaction modulus (K): $p = K \times y$ [units: F/L$^2$] $\iff K = k_h \times B$
- Coefficient of subgrade reaction ($n_h$): rate of increase of subgrade reaction modulus (K) with depth (z): $K = n_h \times z$ [units: F/L$^3$]
Soil reaction (p-y curve) and Horizontal Modulus of Subgrade Reaction ($k_h$)

Net soil reaction per unit length of pile (F/L)

\[ q_h = k_h \cdot y \]

\[ q_h = \frac{p}{B} \]

\[ p = (k_h \times B) \cdot y \]

Units:
- \( q_h = \text{[F/L}^2\text{]} \)
- \( y = \text{[L]} \)
- \( k_h = \text{[F/L}^3\text{]} \)

Careful units of \( k_h \) same as \( k_v \) (F/L³) but genesis is different
Elements of a p-y curve

Soil reaction, \( p \) (F/L) vs. Pile deflection, \( y \) (L)

- \( p = E_{py} \cdot y \)
- \( E_{py-max} \)
- \( P_{ult} \)

\( p - y \) modulus, \( E_{py} \) (F/L^2)

Pile deflection, \( y \) (L)

\( E_{py-max} \)
Elements of a p-y curve

1. Initial slope $E_{py\text{-max}}$:
   - Considerable scatter of reported values.
   - Most $E_{py\text{-max}} = k_h \times B$
   - Some P-Y curves have $E_{py\text{-max}} \to \infty$

2. $P_{\text{ult}}$ (asymptotic value):
   - From ultimate load theories (e.g., Broms, 1964):
     • Clays: $9S_uB$;
     • Sands: $3K_p\sigma'_vB$ or $K_p^2\sigma'_vB$

3. Transition curve(s) from origin to $P_{\text{ult}}$.
Example p-y curves in Sands (Reese et al. 1974)

- $P_{ult} = f(\phi, \text{stress level})$
- $E_{py\text{-max}}$: related to soil stiffness, $B$, etc,
P-Y Curves for Different Soil Types

- **Stiff Clay**: Reese and Welch (1975)
- **Sand**: Reese et al. (1974)
- **API 1987**: (API 1987)
- **Soil w/ c' y φ'**: Reese and Van Impe (2001)
- **Soft Clay**: Matlock (1970)
Experimental P-Y Curves

- Lateral load tests on instrumented piles
- Very few high quality tests are available
- Basis for P-Y Curves proposed in the literature
- Typically from deflected shape measurements (e.g., inclinometers)
- Better if from Moment (or curvature) measurements using closely spaced pairs of strain gages (very few of these)
Additional Instrumentation

![Additional Instrumentation Diagram](image)

- **Front Row**: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
- **Deflection, mm**: 0.0, 20.0, 40.0, 60.0
- **Depth From Top of Pile, m**: 35.6 kN, 48.9 kN, 66.7 kN, 84.5 kN, 97.9 kN
- **Average Load per Pile**: 35.6 kN, 48.9 kN, 66.7 kN, 84.5 kN, 97.9 kN

![Additional Instrumentation Image](image)
P-Y Curves from Experiments

**STEP 1**
Fit a cubic polynomial using the least-squares approach to a 5-node window of deflection data.

\[ y = Ax^3 + Bx^2 + Cx + D \]

**STEP 2**
Differentiate twice the fitted function and multiply it with the EI of the pile to obtain the moment.

\[ y = Ax^3 + Bx^2 + Cx + D \]

\[ M = E_p I_p \frac{d^2y}{dx^2} \]

\[ \frac{d^2y}{dx^2} = 6Ax + 2B \]

\[ M = E_p I_p \left(6Ax + 2B\right) \]

**STEP 3**
Evaluate the linear function at the center-most node (node 3)
P-Y Curves from Experiments

STEP 6
Make an average of the moments obtained on steps 3, 4 and 5, and assign it to the center-most node in step 4

Recall from beam theory:
\[ p = \frac{d^2M}{dx^2} \]
### Commonly used p-y curves for different soils

<table>
<thead>
<tr>
<th>Soil Type and Condition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft clay below the water table</td>
<td>Matlock (1970)</td>
</tr>
<tr>
<td>Stiff clay below the water table</td>
<td>Reese, Cox, and Koop (1975)</td>
</tr>
<tr>
<td>Stiff clay above the water table</td>
<td>Welch and Reese (1972), Reese and Welch (1972)</td>
</tr>
<tr>
<td>Sands</td>
<td>Reese, Cox, and Koop (1974)</td>
</tr>
<tr>
<td>Sands</td>
<td>API RP2A (1991)</td>
</tr>
<tr>
<td>Soils with cohesion and friction</td>
<td>Evans and Duncan (1982)</td>
</tr>
<tr>
<td>Weak rock</td>
<td>Reese (1997)</td>
</tr>
<tr>
<td>Strong rock</td>
<td>Nyman (1982)</td>
</tr>
</tbody>
</table>

(adapted from Reese and Isenhower, 1997)
ANALYTICAL METHODOLOGY
Software for p-y based analysis:

• Solve beam equation with finite difference or finite elements
• COM624
• LPILE
• FB-Pier (FB-Multipier)
• Matlab or Mathcad spreadsheets
Other Methodologies

- Strain Wedge Model
- FEM
- Characteristic Load Method (LPILE based)
POSSIBLE LIMITATIONS OR CHALLENGES
Potential Limitations P-Y Curves

• The soil is idealized as a series of independent nonlinear springs represented by p-y curves. Therefore, the continuous nature of the soil is not explicitly modeled.
Potential Limitations P-Y Curves

• The results are very sensitive to the p-y curves used. **The selection of adequate p-y curves is the most crucial problem when using this methodology to analyze laterally loaded piles** (Reese and Van Impe 2001).

• P-Y curves in literature are empirical in nature. Need to carefully review applicability of the selected curves.
On selection of appropriate p-y modulus and p-y curves

• Important and difficult task.
• Selection of values of initial p-y modulus, $E_{py-max}$, although related to the soil modulus, is also related to the interaction between the pile and the soil.
• Reese and Van Impe (2001) point out that p-y curves and modulus are influenced by several pile related factors, such as:
  • Pile type and flexural stiffness,
  • Type of loading (monotonic or cyclic),
  • Pile geometry,
  • Pile cap conditions, and
  • Pile installation conditions.
Potential Limitations (Continued)

Cross section of pile

- Most P-Y curves only depend on pile width (B). Shape or Depth is not explicitly included in P-Y curves currently in the literature.

\[ p_2 > p_1 > p_3 \]
a) Pilote de Acero y Sección Sólida

\[ I_\phi = 0.0004 \text{ m}^2 \]
\[ E_p = 200 \text{ GPa} \]
\[ f_p = 0.51 \text{ kN.m}^2 \] (constante)

b) Pilote Tubular de Acero con \( t = 0.05 \text{ m} \)

\[ I_\phi = 0.00034 \text{ m}^2 \]
\[ E_p = 200 \text{ GPa} \]
\[ f_p = 6.81 \text{ kN.m}^2 \] (constante)

c) Pilote H de Acero

\[ I_\phi = 0.004 \text{ m}^2 \]
\[ E_p = 200 \text{ GPa} \]
\[ f_p = 8.21 \text{ kN.m}^2 \] (constante)

d) Pilote Hormigón Pretensado

\[ I_\phi = 0.0055 \text{ m}^2 \]
\[ E_p = 26.5 \text{ GPa} \]
\[ f_p = 1.515 \text{ kN.m}^2 \] (no lineal)

e) Pilote de Plástico Reforzado

\[ I_\phi = 0.00034 \text{ m}^2 \]
\[ E_p = 275 \text{ MPa} \]
\[ f_p = 1.272 \text{ kN.m}^2 \] (no lineal)
Potential Limitations (Continued)

Flexural Stiffness (EI) of pile

- P-Y curves don’t directly incorporate effects of EI of pile (Only in pile model).

Ashour and Norris (2000)
The traditional p-y curve (in LPILE) does not account for the pile/shaft EI variation.

Based on the Strain Wedge Model Analysis:

Effect of Pile Bending Stiffness on the p-y Curve in Sand.
EI Effects

Depth

Lateral Deflection

Soil Reaction, $p$ (kN/m)

Lateral Deflection, $y$ (m)

- $p=120$ kN/m
- $p=170$ kN/m

EI

10EI

0.1EI

Depth

$P = 2224$ kN

$P = 4448$ kN

$P = 890$ kN

Reese et al. (1974)

API (1987)
### Route 351 Bridge Case History

#### Table: Geotechnical Test Results

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Description of Material</th>
<th>SPT N&lt;sub&gt;60&lt;/sub&gt;</th>
<th>CPT Tip Resistance q&lt;sub&gt;t&lt;/sub&gt; (Bars)</th>
<th>CPT Sleeve Friction f&lt;sub&gt;s&lt;/sub&gt; (Bars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Silt, silty fine sand (SM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Sand, fine grained, silty, w/ shell fragments, medium dense, brown (SM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Same - grey (SM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Clay, sandy, stiff (CL)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Sand, fine grained, silty, w/ shell fragments, grey (SM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Sand, clayey to silty, medium dense, (SM-SC)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Sand, fine grained, silty, w/ shell fragments, medium dense to dense, grey (SM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Boreholes terminated at 30.6 m depth, Water level 0.9 - 1.5 m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Diagrams:

- **Concrete Core**:
  - Diameter: 10.7 mm
  - Dimensions: 622 mm diameter

- **FRP Tube**:
  - Diameter: 58 mm
  - Dimensions: 89 mm

- **Recycled Plastic Matrix (MDPE)**:
  - Diameter: 43 mm
  - Dimensions: 592 mm diameter

- **Strands**:
  - Diameter: 16 - 12.7 mm
  - Tied to a No. 15M gage wire external spiral with a 0.15 m pitch.

- **Long Rebar**:
  - Diameter: 24 No. 25M
  - Tied to a No. 9M external spiral with a 0.15 m pitch.

  - Welded to a No. 9M internal spiral with a 0.23 m pitch.
Route 351 Bridge Case History

Graph 1: Flexural Stiffness vs. Moment
- Pressed concrete pile
- FRP pile
- Plastic pile

Graph 2: Applied Lateral Load vs. Lateral Deflection
- FRP pile
- Prestressed concrete pile
- Plastic pile
Route 351 Bridge Case History
Route 351 Bridge Case History

The graphs illustrate the relationship between deflection and soil reaction for different materials.

**Graph 1:**
- Soil Reaction, $p$ (kN/m)
- Deflection, $y$ (mm)
- Two curves with different values of $EI$: $1.86 \times 10^5$ kN-m$^2$ and $7.11 \times 10^4$ kN-m$^2$
- Three materials: PSC, FRP, PPI

**Graph 2:**
- Soil Reaction, $p$ (kN/m)
- Deflection, $y$ (mm)
- Two curves with different values of $EI$: $1.86 \times 10^5$ kN-m$^2$ and $7.11 \times 10^4$ kN-m$^2$
- Three materials: PSC, FRP, PPI
Lateral Cyclic loading on Piles

• Limited experimental data.
• API P-Y curves for sands suggest incorporating 10% degradation of p-y curve for offshore piles.
• A few experimental studies developed cyclic P-Y curves
Effect of Cyclic Loading

Deflection (mm)

Load (kN)

1st Cycle Peaks
15th Cycle Peaks
Cyclic P-Y Curves by Little and Briaud (1988):

- Most experiments up to 20 lateral load cycles.
- $y_N = y_1 \cdot N^\alpha$
Cyclic P-Y Curves by Long and Vanneste (1994):

- 34 experiments (some up to 500 lateral load cycles).
- Modified $P_n$ and $Y_n$
  - $P_N = P_1 \cdot N^{-0.4t}$
  - $y_N = y_1 \cdot N^{0.6t}$
Possible limitation with cyclic loading

• Little experience and scarce availability of experimental data.

• Available experiments very few load cycles.

• Wind action on highway signs, sound barrier foundations; Or loading on bridge piles (thermal, current, wave, etc) can involve $N > 10^4$ load cycles during pile design life.
Summary & Conclusions

- The P-Y Curve based methodology for analysis of laterally loaded piles is easy and reliable.
- Empirical in nature, but backed by decades of experience.
- However, several items may still need additional research to overcome some identified possible limitations. (i.e., still room for improvement).
- Also practitioners should be aware of alternative emerging methodologies such as the SWM (Need to incorporate into design tool box) (Several DoT’s already using).
THANK YOU!

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