

# Probabilistic Slope Stability Analysis of an Earth-filled Embankment Dam – Case Study

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## Outline

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- Motivation
- Background – Summary of Analysis Techniques
- Case Study – Methodology
- Case Study – Analysis Results



## Motivation

- Determine the feasibility of implementing probabilistic based design with data from existing projects
- Validate results from probabilistic based design with observed performance

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## Background

Deterministic	Probabilistic
Traditional Standard Based	Non-traditional probability based

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

$$\beta = -\Phi^{-1}[p_f]$$

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## Background

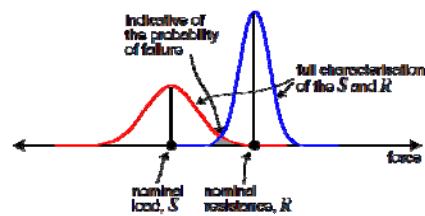
### Deterministic

Traditional Standard Based

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

### Probabilistic

Non-traditional probability based



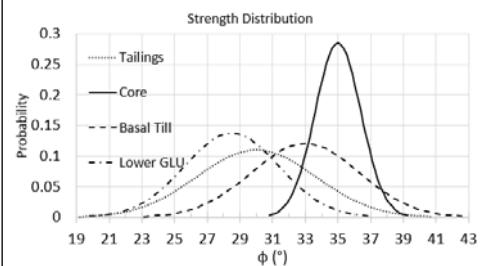
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## Background

### Deterministic

Material Name	Unit Weight (lb/ft <sup>3</sup> )	Cohesion (kPa)	Phi (deg)
Core	131	0	35.0
Tailings	115	0	30.0
Hard Glaciolacustrine	127	0	28.0
Basal Till	134	0	33.0

### Probabilistic



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## Background

Deterministic				Probabilistic			
Material Name	Unit Weight (lb/ft <sup>3</sup> )	Cohesion (kPa)	Phi (deg)				
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The graph displays the probability distribution of unit weight for different materials. The x-axis represents unit weight ( $\gamma$  in lb/ft<sup>3</sup>) from 100 to 150. The y-axis represents Probability from 0 to 0.18. The 'Tailings' material has a low-unit-weight distribution peaking around 115 lb/ft<sup>3</sup>. The 'Core' material has a medium-unit-weight distribution peaking around 130 lb/ft<sup>3</sup>. The 'Basal Till' material has a high-unit-weight distribution peaking around 134 lb/ft<sup>3</sup>. The 'Upper and Lower GLU' material has a very high-unit-weight distribution peaking around 135 lb/ft<sup>3</sup>.

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## Background

Deterministic		Probabilistic	
Traditional Standard Based		Non-traditional probability based	
$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$			

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## Background

### Deterministic

Traditional Standard Based

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

### Probabilistic

Non-traditional probability based

$$\beta = -\Phi^{-1}[p_f]$$

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## Background

### Deterministic

Traditional Standard Based

$$FS = 1.2$$

### Probabilistic

Non-traditional probability based

$$\beta = 2.3$$

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## Background

Deterministic	Probabilistic
Uncertainty handled discretely through the use of “most likely scenarios”, “worst case scenarios”, etc.	The combined effects of various scenarios and their probability of occurrence considered holistically.
Most Likely Case FoS = 1.5	$P_f = 0.5\%$
Worst Case FoS = 1.2	

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## Background

Deterministic	Probabilistic
<b>Analytical</b>	<b>Analytical</b>
$FOS = \frac{\tan \phi}{\tan \alpha}$	$P_f = \int_{g(\phi) \leq 1} f_\phi(\phi) d\phi$
Iteration <ul style="list-style-type: none"> <li>• Limit Equilibrium</li> </ul>	<b>Simulation</b> <ul style="list-style-type: none"> <li>• Monte Carlo simulation</li> </ul>
<b>Approximation</b> <ul style="list-style-type: none"> <li>• Finite element analysis</li> <li>• Tables and charts</li> </ul>	<b>Approximation</b> <ul style="list-style-type: none"> <li>• Tables and charts</li> <li>• Uncertainty propagation</li> <li>• Other reliability methods</li> </ul>

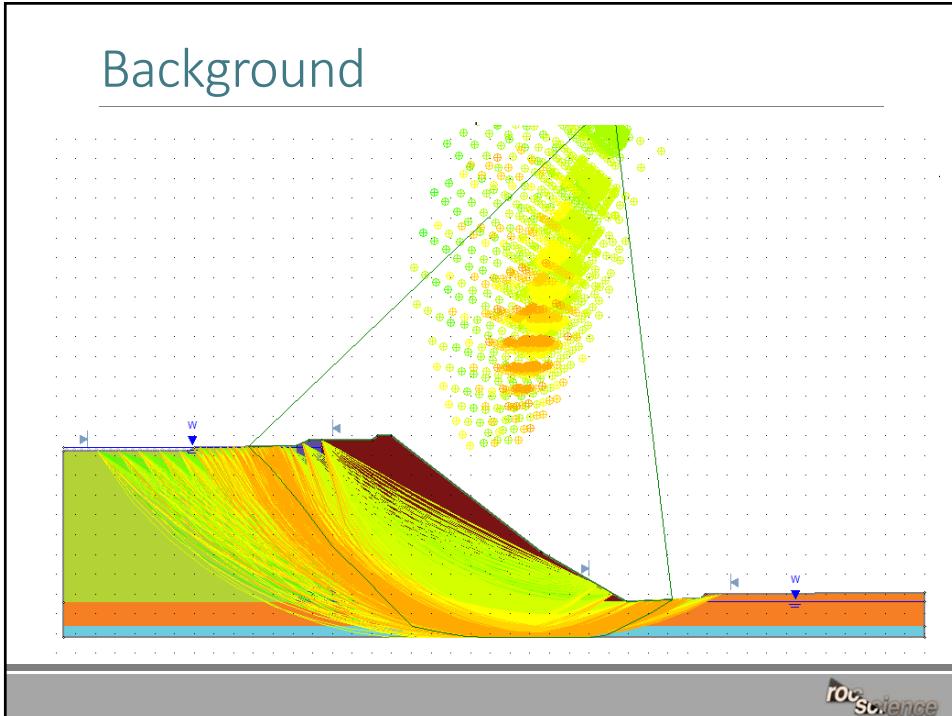
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## Background

Deterministic	Probabilistic
Analytical	Analytical
$FOS = \frac{\tan \phi}{\tan \alpha}$	$P_f = \int_{g(\phi) \leq 1} f_\phi(\phi) d\phi$
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<ul style="list-style-type: none"> <li>• Finite element analysis</li> <li>• Tables and charts</li> </ul>	<ul style="list-style-type: none"> <li>• Tables and charts</li> <li>• Uncertainty propagation</li> <li>• Other reliability methods</li> </ul>

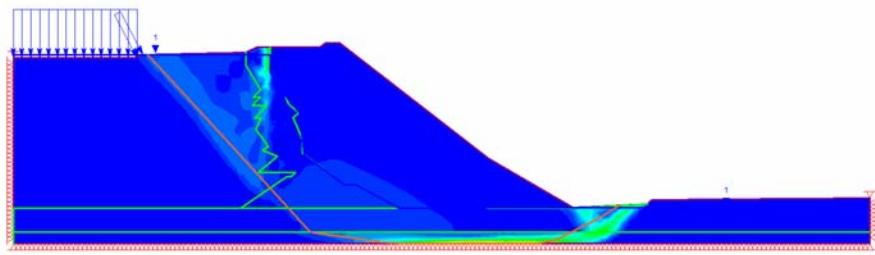
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## Background



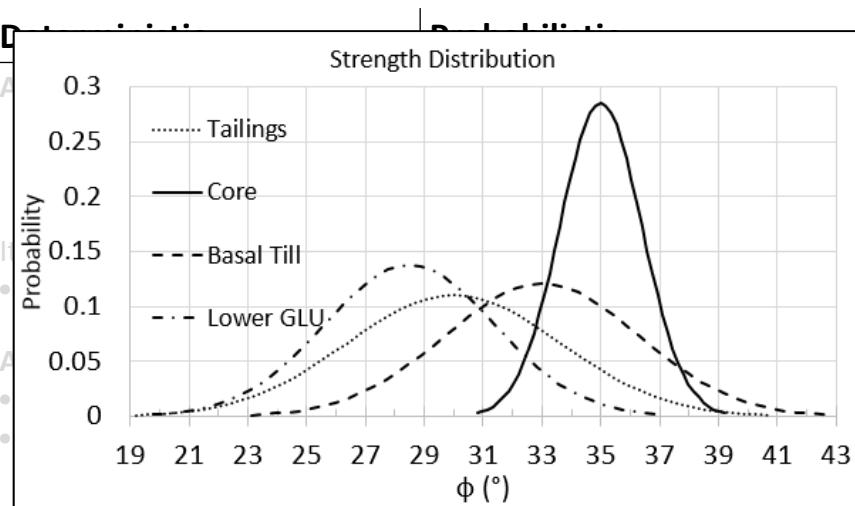
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## Background



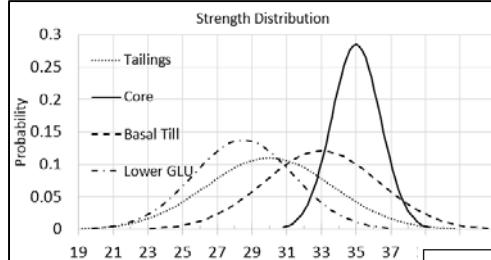
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## Background



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## Background



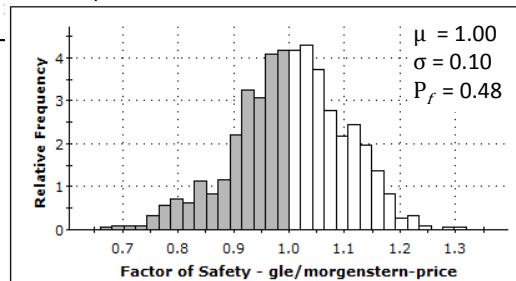
- Limit Equilibrium

### Approximation

- Finite element analysis
- Tables and charts

Probabilistic  
analytical

$$P_f = \int_{g(\phi) \leq 1} f_\phi(\phi) d\phi$$



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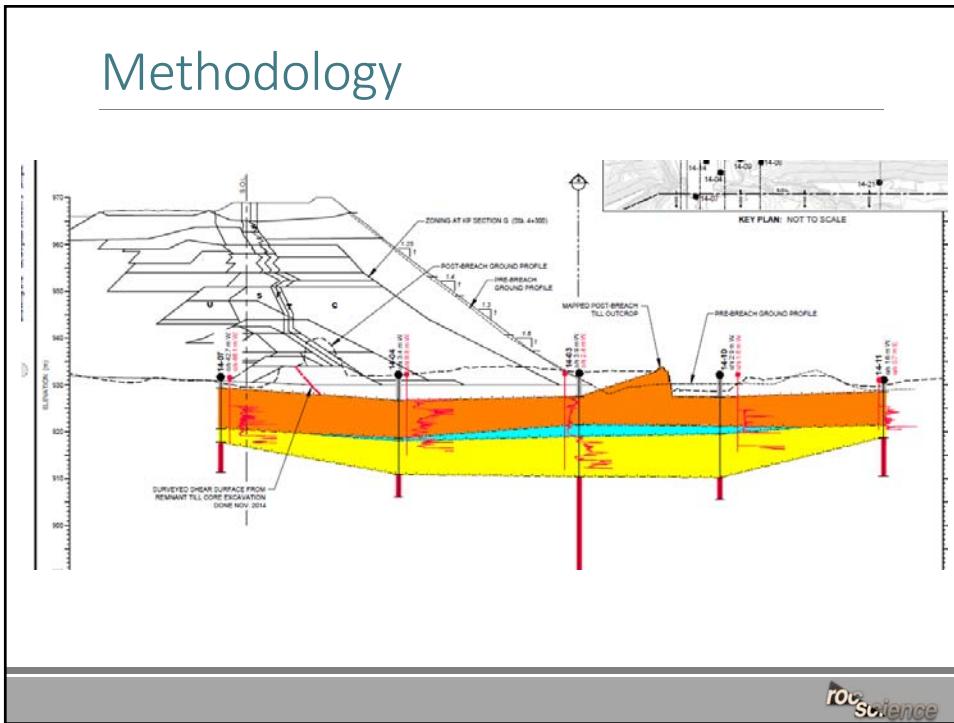
## Case Study



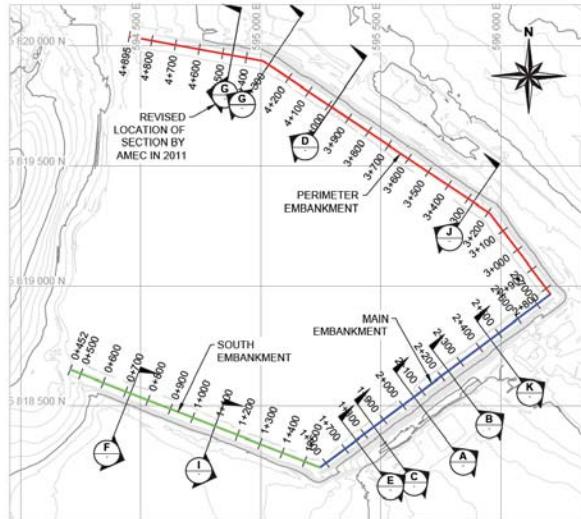
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## Methodology

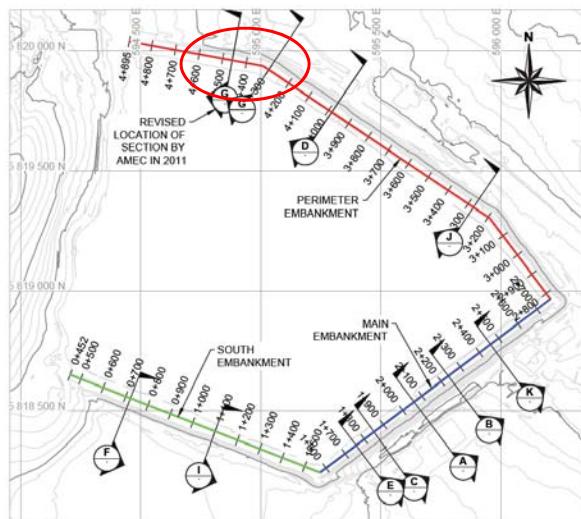


## Methodology



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## Methodology



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## Methodology

**Analytical Model:** Morgenstern-Price 2D Limit Equilibrium

**Reliability Analysis Method:** Monte Carlo Simulation

**Defining the Failure:** Factor of Safety <1

**Defining Material Properties:** Literature review (Phoon & Kulhawy 1999), tailings data from similar mines, calculated some based on lab test data.

**Analysis Tool:** Slide by Rocscience



## Methodology

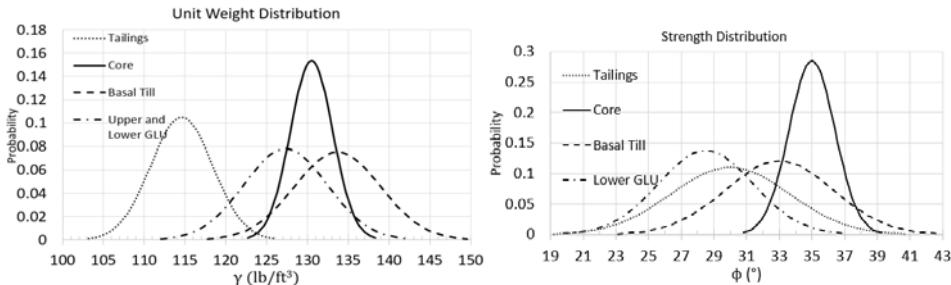
Defining Material Properties:

Material	Unit Weight			Strength				
	Mean (lb/ft <sup>3</sup> )	COV (%)	Standard Deviation	Strength Model	Strength	COV (%)	Standard Deviation	Cohesion (kPa)
Tailings	115	5.0%	5.7	Mohr-Coulomb	30°	12%	3.6°	0
Core	131	3.0%	3.9	Mohr-Coulomb	35°	4%	1.4°	0
Rockfill	140	-	-	Shear/Normal F. Leps Function	-	-	-	-
Upper GLU	127	4.0%	5.1	Stress Ratio	0.24	15%	0.04	0
Lower GLU	127	4.0%	5.1	Mohr-Coulomb	28.5°	9%	2.6°	0
Basal Till	134	4.0%	5.3	Mohr-Coulomb	33°	10%	3.3°	0



# Methodology

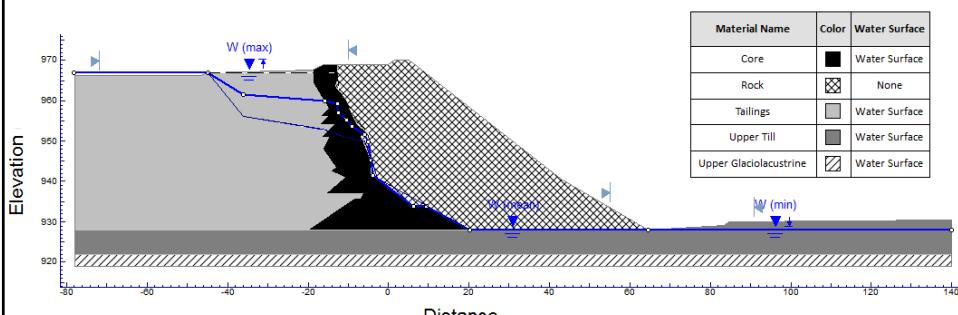
## Defining Material Properties:



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# Methodology

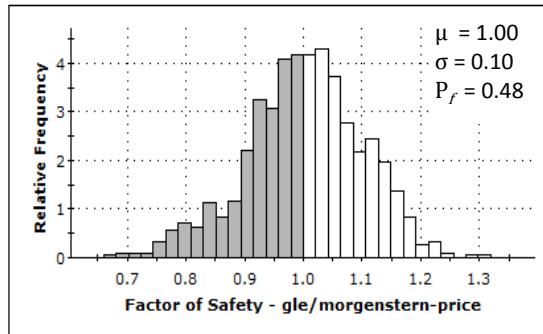
## Model Calibration:



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## Methodology

Model Calibration:

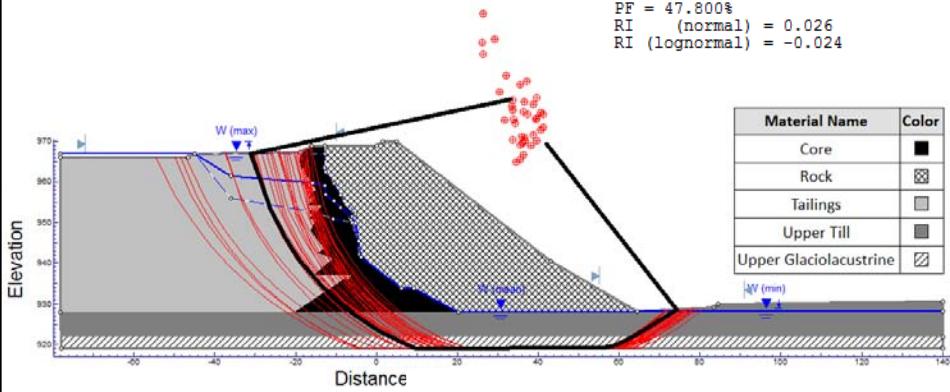


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## Methodology

Model Calibration:

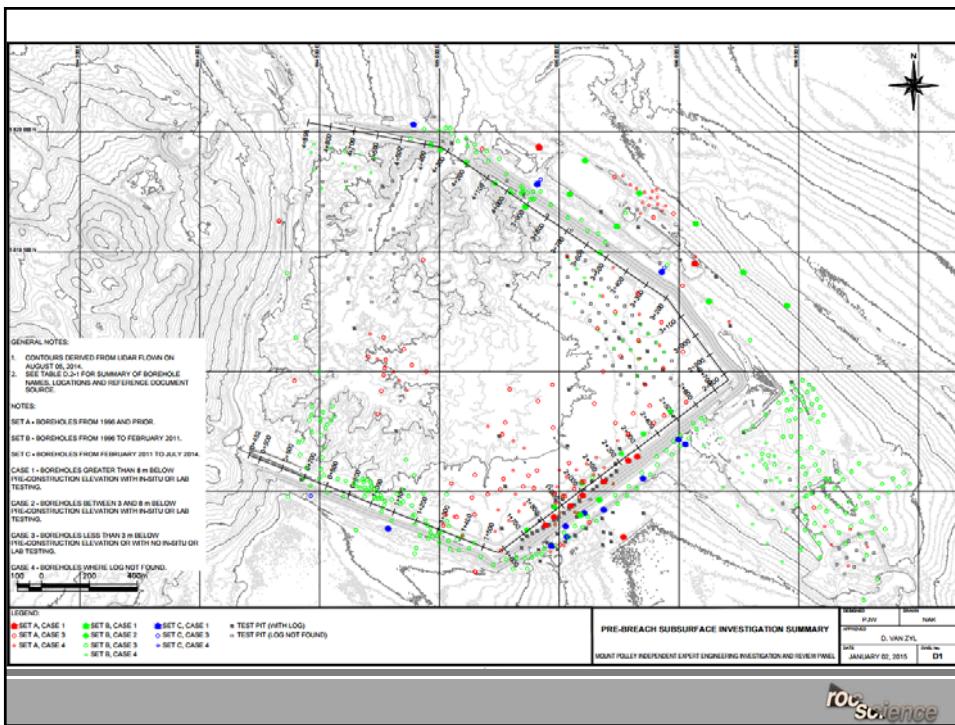
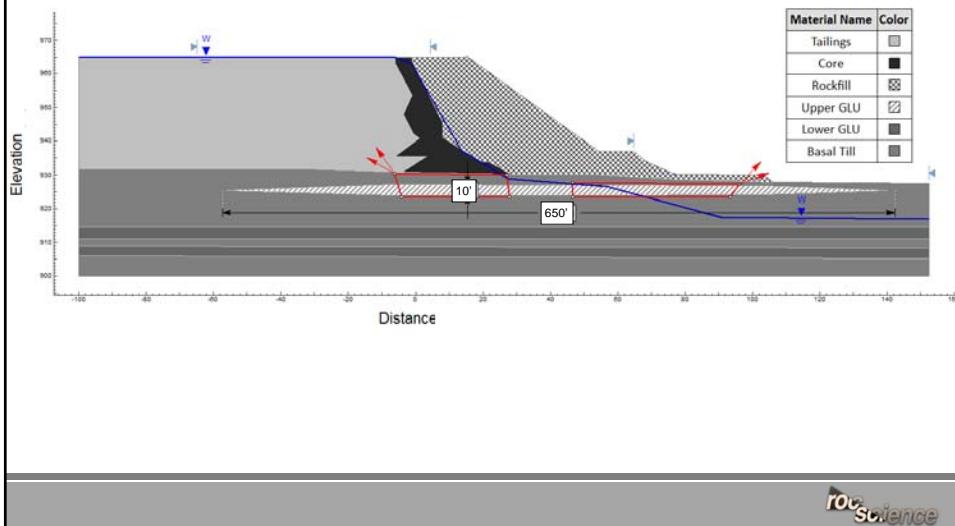
Overall Slope Results  
 FS (mean) = 1.003  
 PF = 47.800%  
 RI (normal) = 0.026  
 RI (lognormal) = -0.024

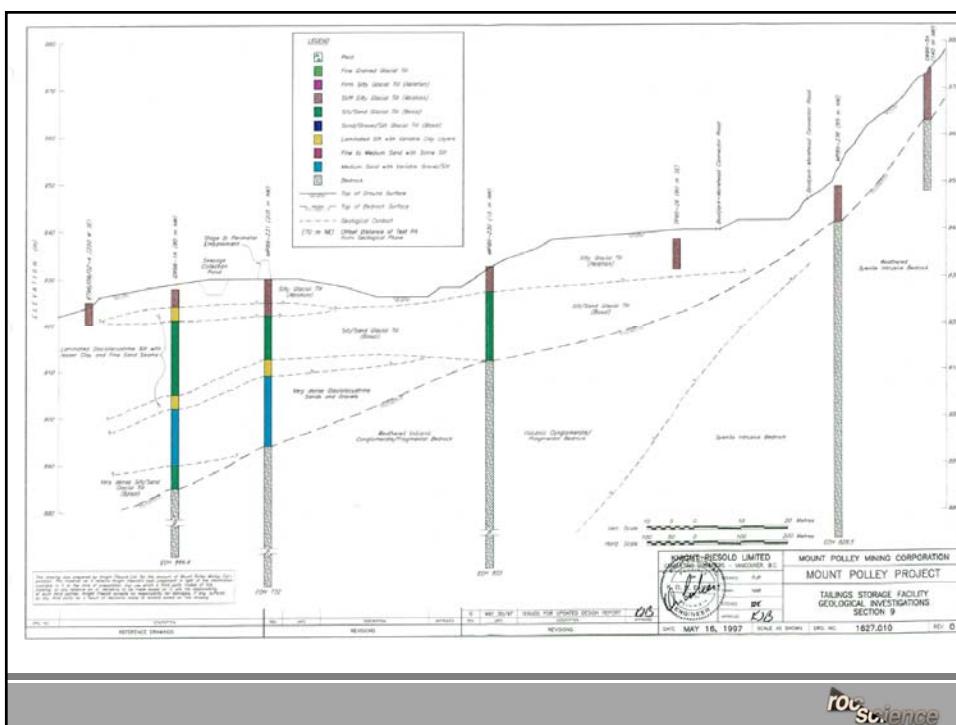
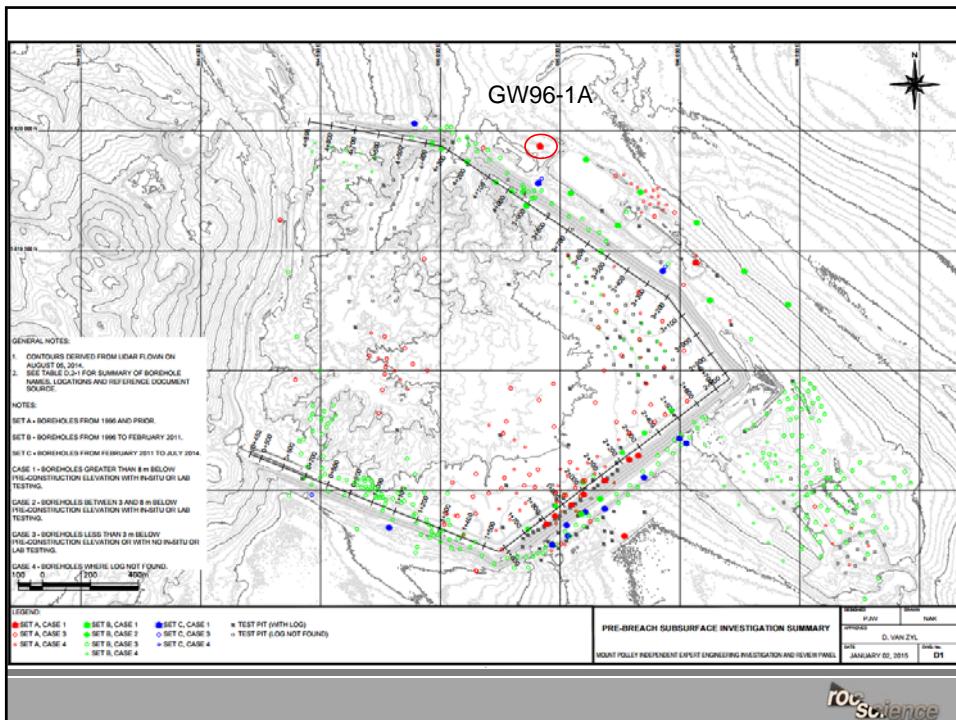


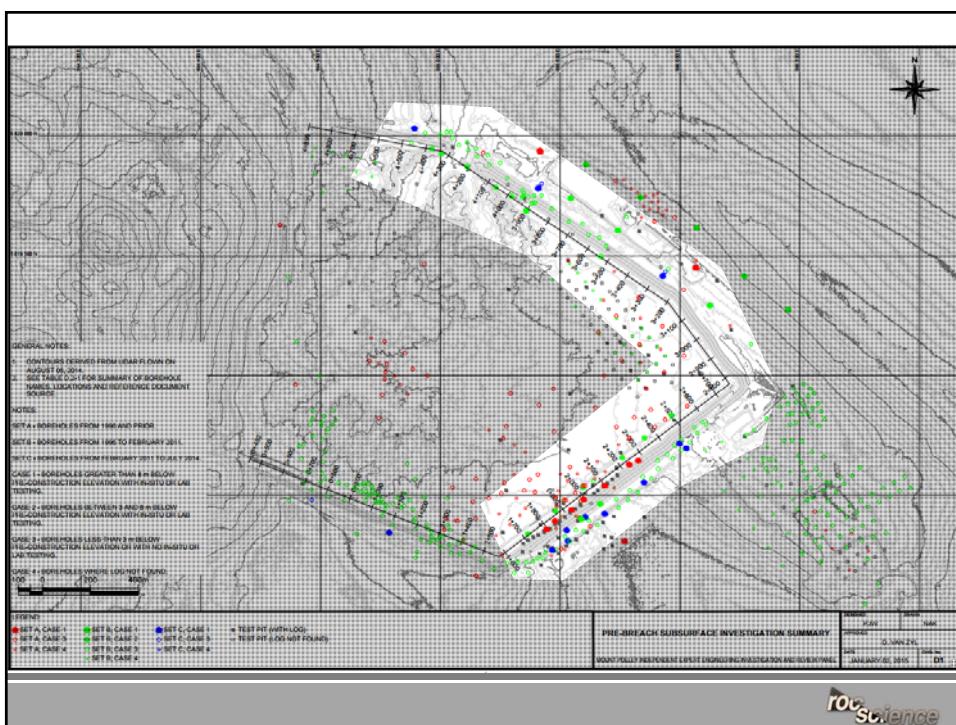
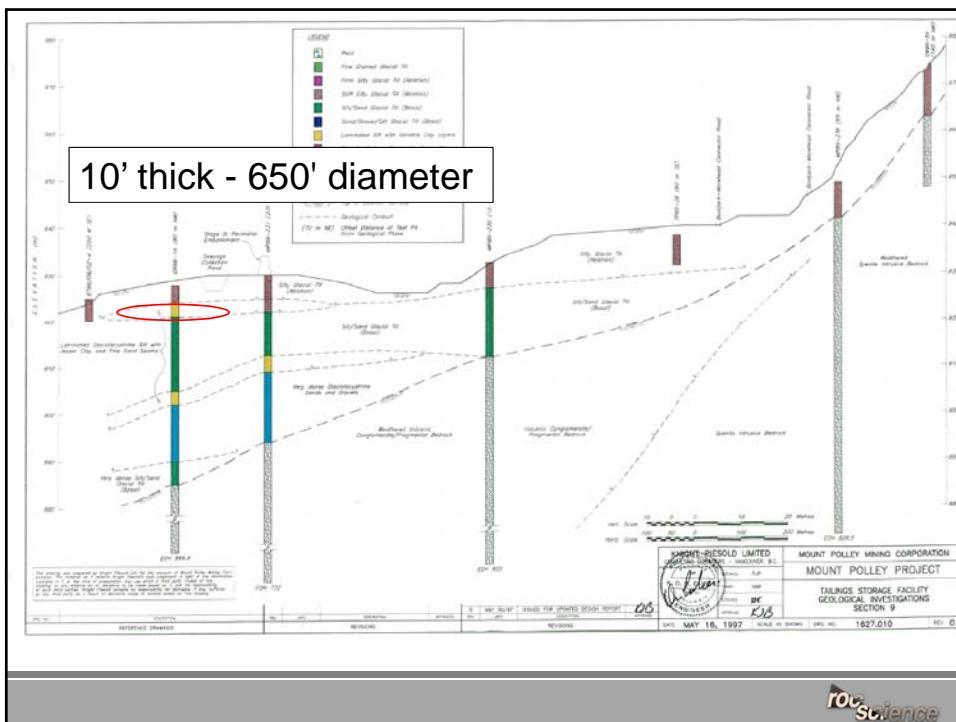
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# Methodology

## Defining Geological Model:







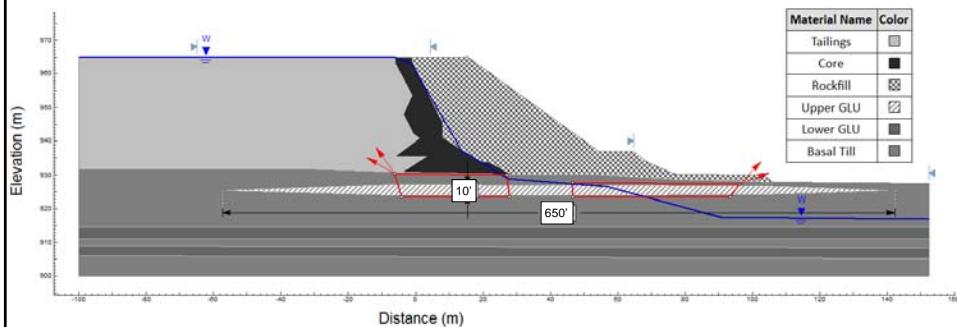
## Methodology

Case	Description	Likelihood (%)
1	No upper GLU	90.3%
2	GLU at 3'-10' depth	0.5%
3	GLU at 10'-20' depth	2.3%
4	GLU at 20'-30' depth	2.3%
5	GLU at 30'-40' depth	2.3%
6	GLU at 40'-50' depth	2.3%

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## Methodology

Defining Geological Model:

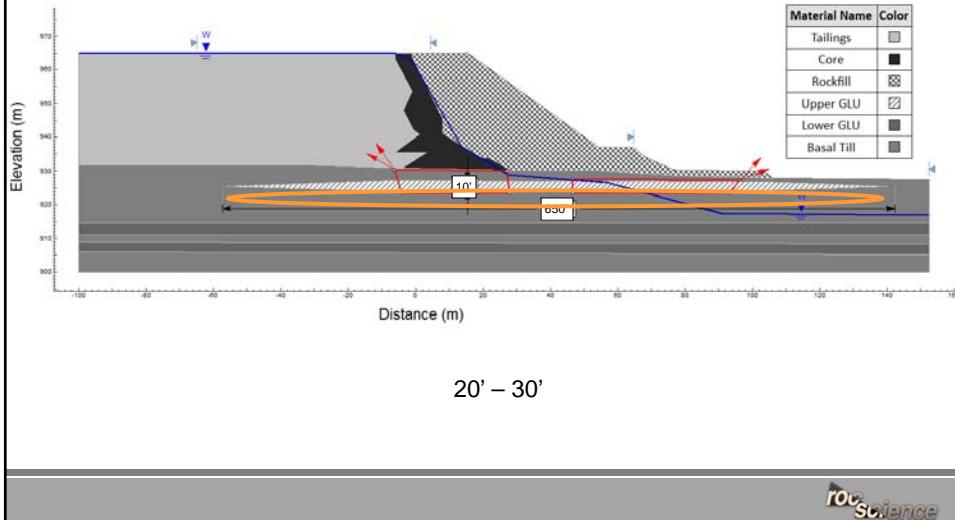


10' – 20'

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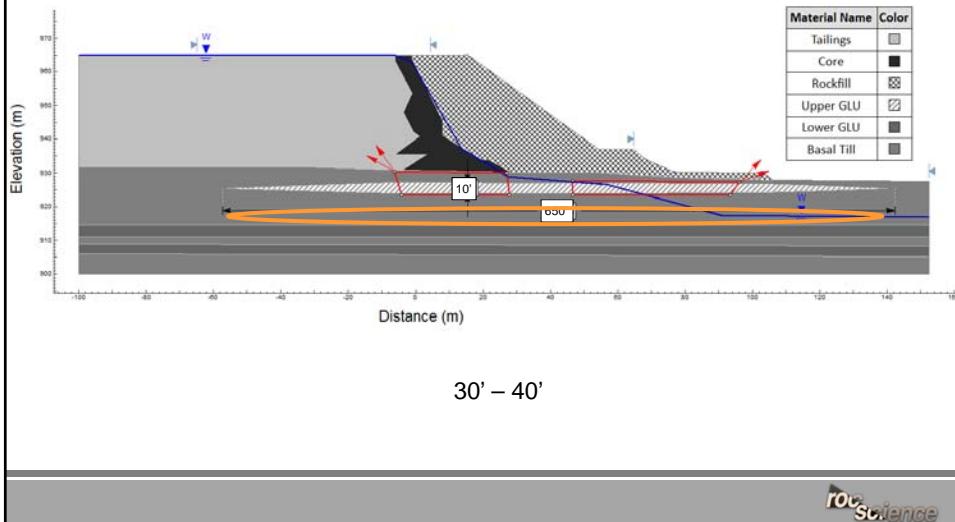
# Methodology

Defining Geological Model:



# Methodology

Defining Geological Model:



## Results

Case	Description	Likelihood ( $p_x$ )	$P_f (x_i)$
1	No upper GLU	90.3%	0.00
2	GLU at 3'-10'	0.5%	0.90
3	GLU at 10'-20'	2.3%	0.84
4	GLU at 20'-30'	2.3%	0.70
5	GLU at 30'-40'	2.3%	0.58
6	GLU at 40'-50'	2.3%	0.25
Weighted probability of slope stability failure			5.90%

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## Conclusion

- Risk = Probability x Consequence
- Discount cash flow valuations when valuing projects for business cases
- Justifying site investigation costs – “You pay for a site investigation program whether you have one or not.” So how many boreholes do we need?

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## Questions?



## Results

Case	Description	Trials	Minutes	Deterministic FS	Factor of Safety						Reliability	
					Mean ( $\mu$ )	Std. Dev. ( $\sigma$ )	Min	Max	Distribution	$\beta$	$P_f$	
Calibration Model	Panel Back-analysis	850	4	1.01	1.00	0.10	0.66	1.30	Normal	0.026	0.48	
	1 No upper GLU	700	3	1.77	1.74	0.15	1.27	2.02	Beta	5.054	2.16E-07	
	2 GLU at 3'-10'	700	10	0.89	0.88	0.11	0.53	1.20	Beta	-1.060	0.85	
	3 GLU at 10'-20'	700	10	0.91	0.90	0.10	0.56	1.22	Normal	-1.004	0.84	
	4 GLU at 20'-30'	700	10	0.95	0.94	0.10	0.58	1.24	Normal	-0.543	0.70	
	5 GLU at 30'-40'	700	10	0.98	0.98	0.11	0.67	1.30	Normal	-0.197	0.58	
	6 GLU at 40'-50'	700	10	1.09	1.08	0.11	0.70	1.45	Normal	0.673	0.25	

